The Maximally-Flat Transmission Bridge

By David W Knight

● Inherent temperature compensation ● Flat amplitude vs. frequency response at the detector ●

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Abstract

The compromises involved in the design of RF bridges using current-transformers often lead to a transformer secondary inductance that is inadequate for good low-frequency amplitude flatness at the detector port. A solution to this problem is to make the transformer secondary loading network into a second-order high-pass filter and choose the components to give either maximal flatness or a small overboost. This approach gives a broadband high-precision current sampling network. To make a transmission bridge, it is necessary to design a corresponding maximally-flat voltage-sampling network. Using passive networks, this can be done using resistive voltage-sampling (RVS) with a compensating inductor in parallel with the lower voltage-sampling arm. The problem of thermal drift in the low-frequency balance condition, due to the variation of transformer core permeability with temperature, is solved by using the same magnetic material for the transformer and the compensating inductor. At high frequencies, the maximally flat impedance bridge degenerates into a conventional RVS bridge. The neutralisation problems associated with this configuration are discussed.

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Introduction

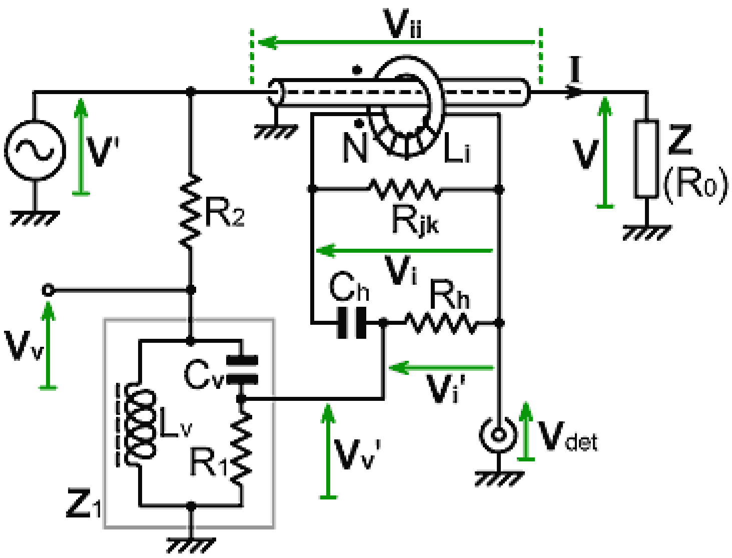
A problem with the design of conventional RF broadband current-transformers lies in reconciling the choice of secondary inductance with the need to maintain good sensitivity at low-frequencies and good phase performance at high-frequencies. This is a particular drawback when trying to design accurate passive directional power-meters or return-loss analysers, because the low-frequency roll-off in detector sensitivity makes a nonsense of any attempt at indicator calibration. In order to obtain amplitude flatness within 1% using a conventional transformer, it is necessary for the secondary reactance to be 7 times greater than the secondary load resistance at the lowest operating frequency[1]. For a lower limit of (say) 1.6 MHz and a load resistance of 50 Ω, this implies an inductance of 35 μH. In HF radio engineering terms, this is a large, and in some cases impractical, inductance. The difficulty becomes apparent when it is observed that a large inductance can be obtained in one of three ways: by using a large number of secondary turns; by using a transformer core with a large magnetic path area; or by using high-permeability core material. None of these options is attractive. Using many turns or a large-area core implies a large conductor length, with attendant problems of propagation delay and winding resistance. Using many turns also implies a low overall transresistance (volts out vs. current in) and hence low sensitivity. Resorting to high-permeability materials (which are more properly intended for EMC filtering and other non-critical applications) results in high core losses, strong dispersion effects (i.e., frequency-dependent inductance variation), and a huge temperature-coefficient of inductance. It starts to look as though good low-frequency performance is a lost cause unless frequency measurement and digital signal processing is included in the design; but there is a passive solution; the *maximally-flat current-transformer network*, which can achieve a flat output without the need for a large secondary inductance.

The theory of the maximally-flat current-transformer was introduced in another article[2] and has been confirmed experimentally[3]. The circuit is that of a conventional current-transformer network with an additional capacitor placed in series with the secondary winding. The capacitor is chosen in relation to the other circuit parameters so that the network becomes a maximally-flat second-order high-pass filter, the effect being to steepen the low-frequency skirt and give an almost-constant in-band amplitude response. The inductance requirement for an in-band amplitude flatness within 2% is reduced by a factor of about 4 by inclusion of this LF-boost capacitor.

In order to use the maximally-flat current-transformer as part of a transmission bridge, it is necessary to devise a companion maximally-flat voltage-sampling network. The point is to tailor the frequency response of the voltage network so that it tracks that of the current-sampling network in both magnitude and phase. Unfortunately, it is not possible to do so by modifying the conventional capacitive potential divider, because the counterpart of the boost capacitor is then a negative resistance. This means that a resistive potential-divider is mandatory if a solution is to be found using passive components. Passive resistive voltage-sampling (RVS) bridges are unsuitable for monitoring high-power transmitters because the divider will typically absorb 1 or 2% of the input power; but it is important to consider the merits of a maximally-flat bridge in the context of an appropriate application. The point is to make an instrument capable of producing accurate return-loss or SWR measurements; in which case the power-level at which the reading is made is a matter of choice, and is preferably low. One virtue of the maximally-flat current-sampling network is that it requires a low number of turns on the transformer and therefore allows a high transresistance. Hence it is ideal for sensitive bridges, i.e., bridges that give an off-balance output of several volts when the transmitted power is in the 10 W to 100 W range.

1 **Current transformers for RF bridges and ammeters**. D W Knight. www.g3ynh.info/zdocs/bridges/
2 **The maximally-flat current transformer**. D W Knight.
3 **Amplitude response of conventional and maximally-flat current transformers**. D W Knight

1. Prototype maximally-flat bridge



Theoretical prototype for a transmission bridge with a maximally-flat amplitude vs. frequency response at the detector port.

The bridge is required to balance (i.e., give zero output at the detector port) when: $\mathbf{Z} = R_0$

The first thing to notice about the prototype circuit above is that it is a low-frequency model. There is no component to represent propagation delay and other effects that mimic transformer secondary parallel capacitance; there is no component to represent the self-capacitance of the LF compensation coil L_v; and it is assumed that the resistor R_2 has no capacitance. Such liberties can be taken at this stage of the analysis because the boost capacitances C_h and C_v are relatively large, and their reactances become correspondingly small at high-frequencies. Thus the bridge degenerates into a conventional resistive voltage-sampling (RVS) arrangement in the upper reaches of its frequency range, allowing the HF neutralising requirements to be deduced by reference to a simplified model.

The current-transformer is shown with a Faraday shield. This is done, not because a shield is necessary, but because it simplifies the circuit analysis[4]. The shield adds stray capacitance from the secondary winding to ground; which, although not desirable, can be lumped with the self-capacitance of L_v in the high-frequency regime. Omitting the shield introduces stray capacitance from the through-line to the secondary winding, which complicates the model considerably; and by reducing the average capacitance per unit length of the through-line, gives rise to a mis-match that manifests itself as an increase in the apparent secondary parallel capacitance of the transformer. Notice also that the shield is earthed on the generator side of the transformer. This means that the capacitive current associated with the load-side part of the shield flows twice through the core, forward on the centre conductor and back on the shield, so that the overall effect on the transformer output is zero.

The current-transformer network differs from that described in the earlier reference[5] by the inclusion of an extra resistance R_{jk}. The reason for the addition is that the Thévenin-equivalent voltage and current sampling networks must be topologically equivalent if they are to have identical phase and magnitude characteristics. In the Thévenin equivalent circuit for the voltage-sampling network, R_2 is in parallel with L_v. Hence we need a resistance in parallel with L_i in order to achieve a frequency-independent solution for the bridge balance condition. Notice that this resistance has a double subscript. This is because it will be resolved eventually into two resistances in parallel: R_j, an actual resistor; and R_k, a resistance to represent the transformer losses. Notice also that, instead of including R_{jk}, we could place a resistance R_{jk} / N^2 across the transformer primary.

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4 **Evaluation and optimisation of current transformer bridges**. D W Knight. See: **Section 19**.

5 **The maximally-flat current transformer**. DWK

1.1 Condition for maximal flatness - current sampling network:

Referring to the circuit diagram given in the previous subsection; we can write an expression for the voltage appearing across the current-transformer secondary ($\mathbf{V}_i$) by first noting that $\mathbf{I} = \mathbf{V}/\mathbf{Z}$ and then applying the ampere-turns rule:

$$\mathbf{V}_i = \mathbf{V}\,\mathbf{Z}_i \,/\, (\,N\,\mathbf{Z}\,) \qquad \mathbf{(1.1.1)}$$

where $\mathbf{Z}_i$ represents the total load across the secondary winding (including the winding itself), i.e.:

$$\mathbf{Z}_i = R_{jk} \,//\, \mathbf{j}X_{Li} \,//\, (\,R_h + \mathbf{j}X_{Ch}\,)$$

(where " // " means "in parallel with"), and N is the transformer turns ratio ($N = N_s / N_p$).
Generally, the number of primary turns $N_p = 1$ for a current-transformer, but there is nothing in principle to prevent the use of several turns of thin coaxial cable.

The high-pass filtered output $\mathbf{V}_i'$ is obtained from $\mathbf{V}_i$ via a potential divider composed of C_h and R_h, thus:

$$\mathbf{V}_i' = \mathbf{V}\; \frac{[\,R_{jk} \,//\, \mathbf{j}X_{Li} \,//\, (\,R_h + \mathbf{j}X_{Ch}\,)\,]}{N\,\mathbf{Z}}\; \frac{R_h}{(\,R_h + \mathbf{j}X_{Ch}\,)} \qquad \mathbf{(1.1.2)}$$

We now need to find the parameter relationship that gives the maximally-flat in-band magnitude response. Since the circuit is a linear network, we can start by dividing both sides of the expression by $\mathbf{V}$ to give the response function in dimensionless form. Inverting the function then allows the parallel combination of impedances to be represented as a series of admittances. Thus:

$$\frac{\mathbf{V}}{\mathbf{V}_i'} = N\,\mathbf{Z} \left[\frac{1}{R_{jk}} + \frac{1}{\mathbf{j}X_{Li}} + \frac{1}{(\,R_h + \mathbf{j}X_{Ch}\,)} \right] \frac{(\,R_h + \mathbf{j}X_{Ch}\,)}{R_h}$$

Multiplying out, and regrouping the terms into reals and imaginaries (noting that $1/\mathbf{j} = -\mathbf{j}$), gives:

$$\frac{\mathbf{V}}{\mathbf{V}_i'} = \frac{N\,\mathbf{Z}}{R_h} \left[\frac{R_h}{R_{jk}} + \frac{X_{Ch}}{X_{Li}} + 1 + \mathbf{j} \left(\frac{X_{Ch}}{R_{jk}} - \frac{R_h}{X_{Li}} \right) \right] \qquad \mathbf{(1.1.3)}$$

To find the reciprocal magnitude response, we take the magnitude of the expression above:

$$\left| \frac{\mathbf{V}}{\mathbf{V}_i'} \right| = \frac{N\,|\mathbf{Z}|}{R_h} \sqrt{ \left[\left(\frac{R_h}{R_{jk}} + \frac{X_{Ch}}{X_{Li}} + 1 \right)^2 + \left(\frac{X_{Ch}}{R_{jk}} - \frac{R_h}{X_{Li}} \right)^2 \right] }$$

Multiplying out gives:

$$\left|\frac{\mathbf{V}}{\mathbf{V}_i'}\right| = \frac{N\,|\mathbf{Z}|}{R_h}\sqrt{\left[\frac{R_h^2}{R_{jk}^2} + \frac{X_{Ch}^2}{X_{Li}^2} + 1 + \frac{2R_h}{R_{jk}} + \frac{2X_{Ch}}{X_{Li}} + \frac{2R_h\,X_{Ch}}{R_{jk}\,X_{Li}} + \frac{X_{Ch}^2}{R_{jk}^2} + \frac{R_h^2}{X_{Li}^2} - \frac{2R_h X_{Ch}}{R_{jk}\,X_{Li}}\right]}$$

which can be rearranged:

$$\left|\frac{\mathbf{V}}{\mathbf{V}_i'}\right| = \frac{N\,|\mathbf{Z}|}{R_h}\sqrt{\left[\left(\frac{R_h}{R_{jk}} + 1\right)^2 + \frac{X_{Ch}^2}{X_{Li}^2} + \frac{X_{Ch}^2}{R_{jk}^2} + \frac{2X_{Ch}}{X_{Li}} + \frac{R_h^2}{X_{Li}^2}\right]} \qquad (1.1.4)$$

The four right-most terms inside the square-root bracket are frequency-dependent. Of those however, the first two, X_{Ch}^2 / X_{Li}^2 and X_{Ch}^2 / R_{jk}^2 will be small within the passband because X_{Ch}^2 diminishes rapidly above the cutoff frequency, X_{Li}^2 increases rapidly, and R_{jk}^2 will be relatively large. This leaves us to consider the last two terms, which can be placed on a common denominator thus:

$$\frac{2X_{Ch}}{X_{Li}} + \frac{R_h^2}{X_{Li}^2} = \frac{2X_{Ch}\,X_{Li} + R_h^2}{X_{Li}^2} = \frac{R_h^2 - 2\,L_i / C_h}{X_{Li}^2}$$

The frequency dependence of the in-band magnitude response can therefore be minimised choosing the circuit parameters so that:

$R_h^2 - 2\,L_i / C_h = 0$

Hence the boost capacitance can be calculated from the expression:

$C_h = 2\,L_i / R_h^2$	**1.1.5**

An alternative version of this expression, which allows X_{Ch} to be eliminated from the network response function once the condition for maximal flatness has been imposed, is:

$X_{Ch} = -\,R_h^2 / 2\,X_{Li}$	**1.1.6**

1.2 Condition for maximal flatness - voltage sampling network

Referring to the circuit diagram in section **1**; the voltage $\mathbf{V_v}$ is derived from an ordinary potential divider and can be written:

$\mathbf{V_v} = \mathbf{V'}\,\mathbf{Z_1} / (R_2 + \mathbf{Z_1})$

i.e.:

$$\mathbf{V_v} = \mathbf{V'} \frac{\mathbf{j}X_{Lv} // (R_1 + \mathbf{j}X_{Cv})}{R_2 + [\,\mathbf{j}X_{Lv} // (R_1 + \mathbf{j}X_{Cv})\,]}$$

Multiplying numerator and denominator by R_2 gives:

$$\mathbf{V_v} = \mathbf{V'} \frac{R_2 // \mathbf{j}X_{Lv} // (R_1 + \mathbf{j}X_{Cv})}{R_2}$$

The high-pass filtered output $\mathbf{V_v}'$ is derived from $\mathbf{V_v}$ via another potential divider, i.e.;

$\mathbf{V_v}' = \mathbf{V_v} R_1 / (R_1 + \mathbf{j}X_{Cv})$

Hence:

$$\mathbf{V_v}' = \mathbf{V'} \frac{R_2 // \mathbf{j}X_{Lv} // (R_1 + \mathbf{j}X_{Cv})}{R_2} \; \frac{R_1}{(R_1 + \mathbf{j}X_{Cv})} \qquad (\mathbf{1.2.1})$$

This expression is exactly analogous to equation (**1.1.2**). Hence, by inspection, the condition for maximal flatness of the voltage-sampling network is:

$C_v = 2\,L_v / R_1^2$	**1.2.2**

This can be re-stated in reactance form as before:

$X_{Cv} = -\,R_1^2 / 2\,X_{Lv}$	**1.2.3**

1.3 Bridge balance conditions

When the bridge is balanced, the arbitrary load impedance **Z** is replaced by the target load resistance R_0. In that condition, the outputs of the current and voltage sampling networks must be equal at all frequencies insofar as the model provides an accurate description of the physical circuit. Using equation (**1.1.2**), we can write a condensed form of the dimensionless current transfer function as:

$$\frac{\mathbf{V_i}'}{\mathbf{V}} = \frac{\mathbf{Z_i}}{N\,\mathbf{Z}} \; \frac{R_h}{(R_h + \mathbf{j}X_{Ch})} \qquad (\mathbf{1.3.1})$$

Where: $\mathbf{Z_i} = R_{jk} // \mathbf{j}X_{Li} // (R_h + \mathbf{j}X_{Ch})$

We can also write a condensed form of the voltage transfer function (**1.2.1**):

$$\mathbf{V_v}' = \mathbf{V}' \; \frac{(R_2 // \mathbf{Z_1})}{R_2} \; \frac{R_1}{(R_1 + \mathbf{j}X_{Cv})}$$

Where $\mathbf{Z_1} = \mathbf{j}X_{Lv} // (R_1 + \mathbf{j}X_{Cv})$

But notice here that **V'** is not the same as **V**. There will be a voltage drop across the transformer primary given by:

$\mathbf{V_{ii}} = \mathbf{I}\,\mathbf{Z_i} / N^2$

i.e., the impedance looking into the current transformer primary will be (to a very good approximation) the secondary load impedance divided by the square of the turns ratio. $\mathbf{V_{ii}}$ can be expressed in terms of **V** by using the substitution:

$\mathbf{I} = \mathbf{V} / \mathbf{Z}$

Hence:

$\mathbf{V}' = \mathbf{V} + \mathbf{V_{ii}} = \mathbf{V}\,[\,1 + \mathbf{Z_i} / (\mathbf{Z}\,N^2)\,]$

Hence, the dimensionless voltage transfer function using the load voltage **V** as the reference is:

$$\frac{\mathbf{V_v}'}{\mathbf{V}} = \frac{(R_2 // \mathbf{Z_1})}{R_2} \; \frac{R_1}{(R_1 + \mathbf{j}X_{Cv})} \left[1 + \frac{\mathbf{Z_i}}{\mathbf{Z}\,N^2} \right] \qquad (\mathbf{1.3.2})$$

The voltage and current transfer functions, both using **V** as the reference level, become equal when $\mathbf{Z} = R_0$. So too do their reciprocals, with the advantage that parallel impedances become sums of admittances. Hence, equating the reciprocals of equations (**1.3.1**) and (**1.3.2**), and moving the primary voltage-drop correction to the current-network side:

$$\frac{R_2}{(R_2 // \mathbf{Z}_1)} \; \frac{(R_1 + \mathbf{j}X_{Cv})}{R_1} = \left[1 + \frac{\mathbf{Z}_i}{R_0 N^2} \right] \frac{N R_0}{\mathbf{Z}_i} \; \frac{(R_h + \mathbf{j}X_{Ch})}{R_h}$$

This, noting that $1/(\mathbf{a}//\mathbf{b}) = (1/\mathbf{a}) + (1/\mathbf{b})$, can be rearranged as follows:

$$\left[\frac{R_2}{\mathbf{Z}_1} + 1 \right] \frac{(R_1 + \mathbf{j}X_{Cv})}{R_1} = \left[\frac{N R_0}{\mathbf{Z}_i} + \frac{1}{N} \right] \frac{(R_h + \mathbf{j}X_{Ch})}{R_h} \qquad (\mathbf{1.3.3})$$

Now notice that as $f \rightarrow \infty$, both X_{Cv} and X_{Ch} vanish, in which case the expression degenerates into the balance condition for a conventional RVS bridge. Observe also that the finite values of C_h and C_v are a matter of free choice; i.e., we do not have to impose the maximal flatness condition, and the amount of response-shaping can be arbitrary within the constraints imposed by the circuit topology. The corollary is that the the frequency tracking of the boost networks, although essential for balance tracking, must also be accomplished regardless of balance considerations. It follows that we must impose the condition:

$(R_1 + \mathbf{j}X_{Cv}) / R_1 = (R_h + \mathbf{j}X_{Ch}) / R_h$

so that both boost networks have the same frequency response. Hence.:

$X_{Cv} / R_1 = X_{Ch} / R_h$

which corresponds to a fixed capacitance ratio:

$C_v / C_h = R_h / R_1$	**1.3.4**

This condition implies that when $\mathbf{V}_v' = \mathbf{V}_i'$, then $\mathbf{V}_v = \mathbf{V}_i$, which means that the overall form of the balance condition for the maximally flat bridge is the same as for the RVS bridge at all frequencies, i.e., substituting **(1.3.4)** into **(1.3.3)**:

$$\frac{R_2}{\mathbf{Z}_1} + 1 = \frac{N R_0}{\mathbf{Z}_i} + \frac{1}{N} \qquad (\mathbf{1.3.5})$$

Now expanding the admittances $1/\mathbf{Z}_1$ and $1/\mathbf{Z}_i$ we get:

$$\frac{R_2}{\mathbf{j}X_{Lv}} + \frac{R_2}{R_1 + \mathbf{j}X_{Cv}} + 1 = \frac{N R_0}{R_{jk}} + \frac{N R_0}{\mathbf{j}X_{Li}} + \frac{N R_0}{R_h + \mathbf{j}X_{Ch}} + \frac{1}{N} \qquad (\mathbf{1.3.6})$$

Every complex expression can be arranged so that it has terms that are purely real and terms that are purely imaginary. When that is done, it can be treated as two separate equalities: that between the reals; and that between the imaginaries. Terms with a complex denominator can be separated by multiplying numerator and denominator by the complex-conjugate of the denominator. Thus equation **(1.3.6)** can be re-written:

$$\frac{R_2}{jX_{Lv}} + \frac{R_2\,(R_1 - jX_{Cv})}{R_1^2 + X_{Cv}^2} + 1 = \frac{N\,R_0}{R_{jk}} + \frac{N\,R_0}{jX_{Li}} + \frac{N\,R_0\,(R_h - jX_{Ch})}{R_h^2 + X_{Ch}^2} + \frac{1}{N}$$

and the real part is:

$$\frac{R_1\,R_2}{R_1^2 + X_{Cv}^2} + 1 = \frac{N\,R_0}{R_{jk}} + \frac{N\,R_h\,R_0}{R_h^2 + X_{Ch}^2} + \frac{1}{N}$$

We can make a further distinction by noting that the expression above has terms that are frequency-dependent and terms that are frequency-independent. It can only be true at all frequencies if the sum of the frequency-independent terms on the left hand side is equal to the sum of the frequency-independent terms on the right hand side (and the same applies to the frequency-dependent terms). Hence we can deduce the requirement:

$$(N\,R_0 / R_{jk}) + 1/N = 1$$

i.e.:

$R_{jk} = R_0\,N^2 / (N-1)$	**1.3.7**

In fact, the reason why R_{jk} was put into the model was so that this equality could be obtained. The only solution for $R_{jk} \rightarrow \infty$ occurs when $N = 1$; i.e., by inspection of **(1.3.6)**, when the $1/N$ term on the right-hand side cancels the 1 on the left-hand side.

Using **(1.3.7)** in **(1.3.6)**, the balance condition now simplifies to:

$$\frac{R_2}{jX_{Lv}} + \frac{R_2}{R_1 + jX_{Cv}} = \frac{N\,R_0}{jX_{Li}} + \frac{N\,R_0}{R_h + jX_{Ch}} \qquad \textbf{(1.3.8)}$$

This relationship must remain true in the limit of infinite frequency; i.e., when $X_L \rightarrow \infty$ and $X_C \rightarrow 0$, hence:

$$R_2 / R_1 = N\,R_0 / R_h \qquad \textbf{(1.3.9)}$$

It can also be seen, by inspection of **(1.3.8)**, that a frequency independent solution exists only when:

$$R_2 / L_v = N\,R_0 / L_i$$

i.e.,

$$L_v / L_i = R_2 / (N\,R_0)$$

and only when:

$$(R_1 + jX_{Cv}) / R_2 = / (R_h + jX_{Ch}) / (N\,R_0)$$

But we already know from **(1.3.9)** that $R_1 / R_2 = R_h / (N\,R_0)$. Hence:

$X_{Cv} / R_2 = X_{Ch} / (N\ R_0)$

i.e.:

$C_h / C_v = R_2 / (N\ R_0)$

Hence, collecting the various relationships:

$$\frac{R_1}{R_h} = \frac{L_v}{L_i} = \frac{C_h}{C_v} = \frac{R_2}{N\ R_0} \qquad (1.3.10)$$

A further important balance relationship comes from equation **(1.3.6)** in the limit where $X_L \rightarrow \infty$ and $X_C \rightarrow 0$:

$$\frac{R_2}{R_1} + 1 = N\ R_0 \left[\frac{1}{R_{jk}} + \frac{1}{R_h} \right] + \frac{1}{N} \qquad (1.3.11)$$

Now let us define a resistance R_{ik} to represent the parallel combination of R_h and R_{jk} ; i.e.:

$R_{ik} = (R_h // R_{jk})$

Substituting this into **(1.3.11)** and subtracting 1 from each side gives:gives:

$$\frac{R_2}{R_1} = \frac{N\ R_0}{R_{ik}} + \frac{1}{N} - 1 \qquad \text{Transformer constant} \qquad (1.3.12)$$

This is the principal voltage-sampling ratio or 'transformer constant'. It appears explicitly in the analysis of the conventional RVS bridge, in which the boost capacitors are shorted-out and the secondary load (including core losses) has degenerated into a single resistance. It will be required in section **3**, where we will use the simplified RVS model as a basis for the high-frequency analysis.

1.4 Low-frequency drop-off

Equations (**1.3.7**) and (**1.3.10**) tell us how to determine component values in the event of arbitrary choices of (say) N , R_2 , R_h and L_i ; but merely having the ability to balance the bridge does not constitute a proper design procedure. We now need to specify the permissible degree of detector sensitivity drop-off at the lowest frequency of operation, and use it to determine the required amount of transformer secondary inductance. To that end, we can start by defining a low-frequency drop-off factor (i.e., the relative magnitude response):

$$\eta_f = \frac{\text{Sensitivity at frequency f}}{\text{Sensitivity at high frequencies}}$$

For the purposes of this comparison, sensitivity is defined as the magnitude of the detector port output when everything except frequency is held constant. Recall also that:

$$|\mathbf{V}_{\mathbf{det}}| = |\mathbf{V}_v' - \mathbf{V}_i'|$$

Hence:

$$\eta_f = \frac{|\ \mathbf{V}_v'{}_{(f)} - \mathbf{V}_i'{}_{(f)}\ |}{|\ \mathbf{V}_v'{}_{(\infty)} - \mathbf{V}_i'{}_{(\infty)}\ |}$$

But the voltage and current sampling networks have the same frequency response. Therefore, for a given degree of mismatch at the load port, $\mathbf{V}_v'$ will remain in constant proportion to $\mathbf{V}_i'$ regardless of frequency. This means that we can evoke a complex constant, **g** say, (where **g** is a function of **Z**) that allows is to write the magnitude response by reference either to the current-sampling network output, or to the voltage-sampling network output, but without the need for both. This assertion can be proved by examining equation (**1.3.1**) and noting that the point in establishing the balance condition is to arrange matters so that:

$$\mathbf{V}_v' = \mathbf{V}\ \{\ \mathbf{Z}_i R_h / [\ N\ (\ R_h + \mathbf{j}X_{Ch}\ )\ ]\ \} / R_0$$

and

$$\mathbf{V}_i' = \mathbf{V}\ \{\ \mathbf{Z}_i R_h / [\ N\ (\ R_h + \mathbf{j}X_{Ch}\ )\ ]\ \} / \mathbf{Z}$$

so that when $\mathbf{Z} \rightarrow R_0$, $\mathbf{V}_v' - \mathbf{V}_i' = 0$

Hence if we define:

$$\mathbf{g} = R_0 / \mathbf{Z}$$

we get:

$$\eta_f = \frac{|\ \mathbf{g}\ \mathbf{V}_i'{}_{(f)} - \mathbf{V}_i'{}_{(f)}\ |}{|\ \mathbf{g}\ \mathbf{V}_i'{}_{(\infty)} - \mathbf{V}_i'{}_{(\infty)}\ |} = \frac{|\ \mathbf{V}_i'{}_{(f)}\ (\ \mathbf{g} - 1)\ |}{|\ \mathbf{V}_i'{}_{(\infty)}\ (\ \mathbf{g} - 1)\ |} = \frac{|\ \mathbf{V}_i'{}_{(f)}\ |}{|\ \mathbf{V}_i'{}_{(\infty)}\ |} \qquad (\mathbf{1.4.1})$$

The reciprocal of the relative magnitude of the current-sampling network output was given earlier as equation (**1.1.4**). Identifying $\mathbf{V}_i'$ as $\mathbf{V}_{i\,(f)}'$, this becomes:

$$\left| \frac{\mathbf{V}}{\mathbf{V}_{i\,(f)}'} \right| = \frac{N\,|\mathbf{Z}|}{R_h} \sqrt{\left[\left(\frac{R_h}{R_{jk}} + 1 \right)^2 + \frac{X_{Ch}^2}{X_{Li}^2} + \frac{X_{Ch}^2}{R_{jk}^2} + \frac{2X_{Ch}}{X_{Li}} + \frac{R_h^2}{X_{Li}^2} \right]} \qquad (\mathbf{1.4.2})$$

where the frequency dependence becomes explicit upon expansion of the reactances. If we apply the condition for maximal flatness (equation **1.1.6**), i.e.:

$X_{Ch} = - R_h^2 / 2\, X_{Li}$

then the last two terms of (**1.4.2**) vanish and we get:

$$\left| \frac{\mathbf{V}}{\mathbf{V}_{i\,(f)}'} \right| = \frac{N\,|\mathbf{Z}|}{R_h} \sqrt{\left[\left(\frac{R_h}{R_{jk}} + 1 \right)^2 + \frac{X_{Ch}^2}{X_{Li}^2} + \frac{X_{Ch}^2}{R_{jk}^2} \right]} \qquad (\mathbf{1.4.3})$$

And if we let $f \rightarrow \infty$, so that $X_L \rightarrow \infty$ and $X_C \rightarrow 0$, we get:

$$\left| \frac{\mathbf{V}}{\mathbf{V}_{i\,(\infty)}'} \right| = \frac{N\,|\mathbf{Z}|}{R_h} \sqrt{\left[\left(\frac{R_h}{R_{jk}} + 1 \right)^2 \right]}$$

This can be simplified by taking the (positive) square-root of the square and noting that

$(R_h + R_{jk}) / R_h R_{jk} = 1 / (R_h // R_{jk})$

Thus:

$$\left| \frac{\mathbf{V}}{\mathbf{V}_{i\,(\infty)}'} \right| = \frac{N\,|\mathbf{Z}|}{R_h // R_{jk}} \qquad (\mathbf{1.4.4})$$

This, of course, is an expression for the relative output of an ideal current-transformer (infinite secondary reactance), where $R_h // R_{jk}$ is the secondary load resistance. Now, to obtain an expression for the drop-off factor (**1.4.1**) (and noting that we are dealing with reciprocal transfer functions), we divide equation (**1.4.4**) by equation (**1.4.3**).

$$\eta_f = \frac{R_h}{R_h // R_{jk}} \Big/ \sqrt{\left[\left(\frac{R_h}{R_{jk}} + 1 \right)^2 + \frac{X_{Ch}^2}{X_{Li}^2} + \frac{X_{Ch}^2}{R_{jk}^2} \right]} \qquad (\mathbf{1.4.5})$$

This tells us that the load impedance $\mathbf{Z}$ makes no difference to the frequency response. The turns ratio of the current transformer (N) does make a difference however, even though it does not appear explicitly; firstly, because the number of secondary turns (N_s) dictates L_i once the transformer core has been selected; and secondly, because N determines R_{jk} according to equation (**1.3.7**).

$R_{jk} = R_0 N^2 / (N-1)$

Notice also that:

$1 + R_h / R_{jk} = R_h / (R_h // R_{jk})$

so that $\eta_f \to 1$ as $f \to \infty$.

What we require for design purposes however, is to be able to specify a drop-off factor and find the frequency at which it occurs. This will enable us to adjust the circuit parameters (particularly L_i) until the specified maximum drop-off is achieved *at or below* the minimum required working frequency. This entails solving (**1.4.5**) for f , with η_f as an independent (input) variable. We start by squaring (**1.4.5**) and taking the reciprocal:

$$\frac{1}{\eta_f^2} = 1 + \left[\frac{X_{Ch}^2}{X_{Li}^2} + \frac{X_{Ch}^2}{R_{jk}^2} \right] \Big/ \left[\frac{R_h}{R_h // R_{jk}} \right]^2$$

It will simplify matters from now on if we use the substitution:

$R_{ik} = (R_h // R_{jk})$

Where R_{ik} (introduced earlier) represents the total resistive load on the transformer in the high frequency limit. Thus:

$$\frac{1}{\eta_f^2} - 1 = \frac{R_{ik}^2}{R_h^2} \left[\frac{X_{Ch}^2}{X_{Li}^2} + \frac{X_{Ch}^2}{R_{jk}^2} \right]$$

The number of variables can also be reduced by using the maximal flatness condition (**1.1.6**) as a substitution, i.e.:

$X_{Ch} = - R_h^2 / 2 X_{Li}$

Thus:

$$\frac{1}{\eta_f^2} - 1 = \left[\left(\frac{R_h^2}{2X_{Li}^2} \right)^2 + \left(\frac{R_h^2}{2X_{Li} R_{jk}} \right)^2 \right] \frac{R_{ik}^2}{R_h^2}$$

This can be put into standard form:

$$\frac{R_h^2}{4X_{Li}^4} + \frac{R_h^2}{4X_{Li}^2\,R_{jk}^2} - \left(\frac{1}{\eta_f^2} - 1 \right) \frac{1}{R_{ik}^2} = 0$$

which shows that it is a quadratic equation in $(1 / X_{Li})^2$. In this case, the process of solving it will be assisted by multiplying throughout by $4 / R_h^2$:

$$\frac{1}{X_{Li}^4} + \frac{1}{X_{Li}^2\,R_{jk}^2} - \frac{4}{R_h^2\,R_{ik}^2} \left(\frac{1}{\eta_f^2} - 1 \right) = 0$$

Hence $a = 1$, $b = 1 / R_{jk}^2$ and $c = -[4/ (R_h^2\,R_{ik}^2)]\,[(1/\eta_f^2) - 1]$

and the solution is:

$(1/ X_{Li})^2 = [-b \pm\sqrt{(b^2 - 4ac)}] / 2a$

i.e.:

$$\frac{1}{X_{Li}^2} = \frac{-1}{2R_{jk}^2} \pm \tfrac{1}{2} \sqrt{\left[\frac{1}{R_{jk}^4} + \frac{16}{R_h^2\,R_{ik}^2} \left(\frac{1}{\eta_f^2} - 1 \right) \right]}$$

This has two solutions; but $1 / X_{Li}^2$ is positive, and the only way in which a positive right-hand side can be obtained is by taking the positive square root. Hence (also multiplying ½ into the square-root bracket):

$$\frac{1}{X_{Li}^2} = \frac{-1}{2R_{jk}^2} + \sqrt{\left[\frac{1}{4R_{jk}^4} + \frac{4}{R_h^2\,R_{ik}^2} \left(\frac{1}{\eta_f^2} - 1 \right) \right]}$$

Now let us identify

$X_{Li} = 2\pi f_\eta\, L_i$

where f_η is the lower frequency limit at which the output has diminished by a factor of η_f. Hence:

$$f_\eta = 1 \Big/ 2\pi\, L_i \sqrt{\left\{ \frac{-1}{2R_{jk}^2} + \sqrt{\left[\frac{1}{4R_{jk}^4} + \frac{4}{R_h^2\,R_{ik}^2} \left(\frac{1}{\eta_f^2} - 1 \right) \right]} \right\}} \qquad \textbf{(1.4.6)}$$

The equation above tells us that the lower frequency limit is inversely proportional to the transformer secondary inductance.

Now, recalling that $R_{ik} = (R_h // R_{jk})$, notice that when $R_{jk} \to \infty$, $R_{ik} \to R_h$. In that limit, equation (**1.4.6**) reduces to:

$$f_\eta = \frac{R_h}{(2\sqrt{2})\pi\, L_i^{\,4} \sqrt{[\, (1/\eta_f^2) - 1 \,]}} \qquad (\mathbf{1.4.7})$$

This expression is still a fair approximation to f_η because it is intended that R_{jk} should be relatively large. More importantly however, it brings out the major influences, which are that f_η can be reduced either by increasing L_i or by reducing R_h.

Given that R_{jk} is finite however, equation (**1.4.6**) falls into its most convenient form when we forcibly remove the factor $1/(4R_{jk}^{\,4})$ from the second square-root bracket, and then remove the factor $1/(2R_{jk}^{\,2})$ from the first square root bracket. This operation (noting that $2/\sqrt{2} = \sqrt{2}$) gives:

$$\boxed{f_\eta = R_{jk} \Big/ (\sqrt{2})\pi\, L_i \sqrt{\left[\, -1 + \sqrt{\left(1 + \frac{16\, R_{jk}^{\,4}\, [\, (1/\eta_f^2) - 1]}{R_h^{\,2}\, R_{ik}^{\,2}} \right)} \,\right]} \qquad (\mathbf{1.4.8})}$$

where:

$R_{ik} = (R_h // R_{jk})$

and (from equation **1.3.7**):

$R_{jk} = R_0 N^2 / (N - 1)$

1.5 Evaluating candidate transformers

The secondary inductance of the current transformer is derived from the number of turns according to the expression:

$L_i = A_L N_s^2$

where A_L is the inductance factor of the core (either taken from the manufacturer's data sheet or preferably measured), and N_s is the number of secondary turns. Also, from equation **(1.3.7)**, we have:

$R_{jk} = R_0 N^2 / (N-1)$

We can use these substitutions in **(1.4.8)** to obtain an expression that can be used to determine the number of turns that must be wound on a given transformer core in order to achieve a desired low-frequency limit. For a transformer with a 1-turn primary, where $N_s = N$, we get:

$$f_\eta = R_0 \Big/ (\sqrt{2})\pi A_L (N-1) \sqrt{\left[-1 + \sqrt{\left(1 + \frac{16 R_0^4 N^8 [(1/\eta_f^2) - 1]}{(N-1)^4 R_h^2 R_{ik}^2} \right)} \right]} \qquad (\mathbf{1.5.1})$$

If the transformer has more than one turn in the primary, f_η is multiplied by a factor:

$N^2 / N_s^2 = (1/ N_p)^2$

This might make it look as though adding more turns to the primary is beneficial, but in fact, increasing N_p reduces N and causes f_η to increase slightly overall. Hence there is probably no advantage in using a multi-turn primary unless very high sensitivity (very low N) is required. (For more discussion of multi-turn primaries see section **1.10**).

Equation **(1.5.1)** can be made more tractable for calculation purposes by using the identity:

$R_{ik} = (R_h // R_{jk})$

i.e., if we adopt R_{ik} (the total resistive load on the current-transformer) as a principal design parameter, we can eliminate R_h using:

$1 / R_h = (1 / R_{ik}) - (1 / R_{jk})$

Substituting for R_{jk} using **(1.3.7)** we get:

$1 / R_h = (1 / R_{ik}) - [(N-1) / (R_0 N^2)]$

which can be put on a common denominator:

$1 / R_h = [R_0 N^2 - (N-1) R_{ik}] / (R_{ik} R_0 N^2)$

Using this substitution in **(1.5.1)** gives:

$$f_\eta = R_0 \Bigg/ (\sqrt{2})\pi A_L (N-1) \sqrt{\left[-1 + \sqrt{\left(1 + \frac{16 R_0^2 N^4 [R_0 N^2 - (N-1)R_{ik}]^2 \; [(1/\eta_f^2) - 1]}{(N-1)^4 R_{ik}^4} \right)} \right]}$$

which can be rearranged to give:

$$f_\eta = R_0 \Bigg/ (\sqrt{2})\pi A_L (N-1) \sqrt{\left\{ -1 + \sqrt{\left[1 + \frac{[(1/\eta_f^2) - 1]\, 16 N^4 R_0^2}{(N-1)^2 R_{ik}^2} \left(\frac{N^2 R_0}{(N-1) R_{ik}} - 1 \right)^2 \right]} \right\}}$$

Noting the recursive nature of the last term in the inner square-root bracket, this expression can be put into a form suitable for two-step calculation. First we define, say:

$$U = N^2 R_0 / [(N-1) R_{ik}] \qquad \text{. . . . . (1.5.3)}$$

$$= R_{jk} / R_{ik}$$

$$= 1 + R_{jk} / R_h$$

Then:

$$\boxed{f_\eta = \frac{R_0}{(\sqrt{2})\pi A_L (N-1) \sqrt{\{ -1 + \sqrt{[1 + [(1/\eta_f^2) - 1] 16 U^2 (U-1)^2] } \}}} \qquad \text{(1.5.2)}}$$

1.6 Quadrature point

Since the maximally-flat voltage and current sampling networks are second-order high-pass filters, their outputs at frequencies below the working range can be phase-shifted (relative to the generator) by more than +90°. Hence, at frequencies below the point at which +90° quadrature occurs, any phase analysis carried out will require that 180° is added to the result returned by an inverse-tangent function-call.

For the special case when $R_{jk} \to \infty$, the quadrature point coincides with the lower -3 dB point. This is easy to demonstrate by noting that the relative voltage output of a network at the -3 dB point is $1/\sqrt{2}$. When $\eta_f = 1/\sqrt{2}$, $(1/\eta_f^2) - 1 = 1$. Putting this into equation **(1.4.7)**, and allocating the symbol f_x to the phase-crossover frequency, we get:

$$f_x = R_h / \{ (2\sqrt{2})\pi \, L_i \}$$

where, from equation **(1.1.5)**;

$$R_h = \sqrt{(2 \, L_i / C_h)}$$

Hence:

$$f_x = 1/ [\, 2\pi \sqrt{(L_i C_h)} \,]$$

For the general case when R_{jk} is finite, the quadrature-point frequency can be determined from the reciprocal current-transformer response function:

$$\frac{\mathbf{V}}{\mathbf{V_i'}} = \frac{N\,\mathbf{Z}}{R_h} \left[\frac{R_h}{R_{jk}} + \frac{X_{Ch}}{X_{Li}} + 1 + \mathbf{j} \left(\frac{X_{Ch}}{R_{jk}} - \frac{R_h}{X_{Li}} \right) \right]$$

given earlier as **(1.1.3)**

The phase angle φ (say) of a phasor in the form $a+\mathbf{j}b$ is given by $\mathrm{Tan}\varphi = b/a$. Here however, we have the reciprocal of a relative voltage, i.e. a phasor in the form:

$$1 / (a + \mathbf{j}b) = (a - \mathbf{j}b) / (a^2 + b^2)$$

Hence the phase tangent in this case is given by: $\mathrm{Tan}\varphi = -b/a$. Notice also that **(1.1.3)** is not quite in the $a+\mathbf{j}b$ form because the main load impedance **Z** is complex. Since the tangent is a ratio however, **Z** is cancelled-out; which tells us that the network phase response (as distinct from the actual phase of the output) is not affected by the load. Thus the phase response is given by:

$$\mathrm{Tan}\varphi = \left[\frac{R_h}{X_{Li}} - \frac{X_{Ch}}{R_{jk}} \right] \Big/ \left[\frac{R_h}{R_{jk}} + 1 + \frac{X_{Ch}}{X_{Li}} \right]$$

The quadrature point occurs when $\mathrm{Tan}\varphi \to \infty$, i.e., when the denominator of the expression above goes to zero. Hence, at the quadrature point:

$$\frac{R_h}{R_{jk}} + 1 + \frac{X_{Ch}}{X_{Li}} = 0$$

Where $X_{Li} = 2\pi f_x L_i$
and $X_{Ch} = -1/(2\pi f_x C_h)$

Hence, expanding the reactances and rearranging:

$f_x = 1/ \{ 2\pi \sqrt{[L_i C_h (1 + R_h / R_{jk})]} \}$	**1.6.1**

and the phase angle can be calculated using:

$$\varphi = \text{Arctan} \left[\left(\frac{R_h}{X_{Li}} - \frac{X_{Ch}}{R_{jk}} \right) \Big/ \left(\frac{R_h}{R_{jk}} + 1 + \frac{X_{Ch}}{X_{Li}} \right) \right] + n \times 180 \qquad \text{[degrees]}$$

Where $n = 0$ when $f > f_x$, and $n = 1$ when $f < f_x$.

1.7 Preliminary design calculations

Shown below is a snapshot of a spreadsheet calculation (**maxflat_prelim.ods**). This determines the number of turns that must be wound on a transformer core having an A_L of 67 nH/turn^2 in order to meet various low-frequency drop-off criteria (a single-turn primary winding is assumed). The A_L value was selected on the basis that the Amidon[6] FT50-61 core has a published A_L of 68.8 nH/turn^2, but small toroidal transformers typically have a leakage inductance of about 1% to 2%; so the coupled secondary inductance of the transformer will be about 0.98 of the total inductance. Since the published A_L has a tolerance of ±25%, there is no point in resorting to decimal places. It is, of course, best to give A_L as 0.98 of an actual measurement, but the published value has to suffice until the experimental work begins.

The only other input parameters required for the part of the calculation shown are the bridge target load resistance (R_0 = 50 Ω, as usual), and the total resistive secondary load (R_{ik}) for which 50 Ω is also a reasonable starting value.

The calculation tells us that for cores in the middle of the tolerance range, and assuming a working frequency range of 1.6 MHz and above; the LF drop-off can be kept within 1% by using a 12-turn transformer (with a small downward adjustment of R_{ik}), or within 2% by using an 11-turn transformer, or within 5% by using a 10-turn transformer.

Notice incidentally, that the -3 dB frequency is slightly different from the quadrature frequency (f_x) due to the finite value of R_{jk}.

Input parameters:	R_0 = 50 Ω	R_{ik} = 50 Ω	A_L = 67 nH / turn²

η_r			0.99	0.98	0.95	-3dB				
$(1/\eta_r^2)-1$			0.02	0.04	0.11	1				
N	Li	U	fη	fη	fη	fη	fx	Rjk	Rh	Ch
	/ μH		/ MHz	/ MHz	/ MHz	/ MHz	/ MHz	/ Ω	/ Ω	/ pF
4	1.072	5.33	16.02293	13.27151	10.32582	5.85487	5.82329	266.67	61.54	566.15
5	1.675	6.25	9.97310	8.28811	6.46792	3.67938	3.66539	312.50	59.52	945.50
6	2.412	7.20	6.79211	5.65647	4.42261	2.52107	2.51402	360.00	58.06	1430.82
7	3.283	8.17	4.92018	4.10322	3.21219	1.83356	1.82965	408.33	56.98	2022.58
8	4.288	9.14	3.72730	3.11138	2.43782	1.39284	1.39051	457.14	56.14	2721.04
9	5.427	10.13	2.92095	2.43994	1.91291	1.09366	1.09219	506.25	55.48	3526.35
10	6.700	11.11	2.35058	1.96448	1.54085	0.88138	0.88040	555.56	54.95	4438.62
11	8.107	12.10	1.93236	1.61557	1.26761	0.72535	0.72468	605.00	54.50	5457.90
12	9.648	13.09	1.61661	1.35198	1.06108	0.60735	0.60687	654.55	54.14	6584.24
13	11.323	14.08	1.37239	1.14801	0.90119	0.51594	0.51559	704.17	53.82	7817.67
14	13.132	15.08	1.17963	0.98695	0.77488	0.44371	0.44345	753.85	53.55	9158.22
15	15.075	16.07	1.02481	0.85755	0.67338	0.38565	0.38545	803.57	53.32	10605.89
16	17.152	17.07	0.89860	0.75203	0.59059	0.33827	0.33812	853.33	53.11	12160.71

6 http://www.amidoncorp.com/

Formulae used (see open document spreadsheet file **maxflat_prelim.ods**)	Equation
$L_i = A_L N^2$	
$U = N^2 R_0 / [(N-1) R_{ik}]$	**1.5.3**
$f_\eta = R_0 / [(\sqrt{2})\pi A_L (N-1) \sqrt{\{ -1 + \sqrt{[1 + [(1/\eta_f^2) - 1] 16 U^2 (U-1)^2]} \}}]$	**1.5.2**
$R_{jk} = R_0 N^2 / (N-1)$	**1.3.7**
$R_h = R_{jk} R_{ik} / (R_{jk} - R_{ik})$	
$C_h = 2 L_i / R_h^2$	**1.1.5**
$f_x = 1/ \{ 2\pi \sqrt{[L_i C_h (1 + R_h / R_{jk})]} \}$	**1.6.1**

1.8 Losses in the current-sampling network

It was mentioned earlier, that the resistance R_{jk} directly in parallel with the current-transformer secondary winding was given a double subscript because it is a combination of resistive losses in the transformer core (R_k) and an actual resistor to make up the difference (R_j); i.e.:

$R_{jk} = R_j // R_k$

This means that we need to make an estimate for R_k in order to determine R_j .

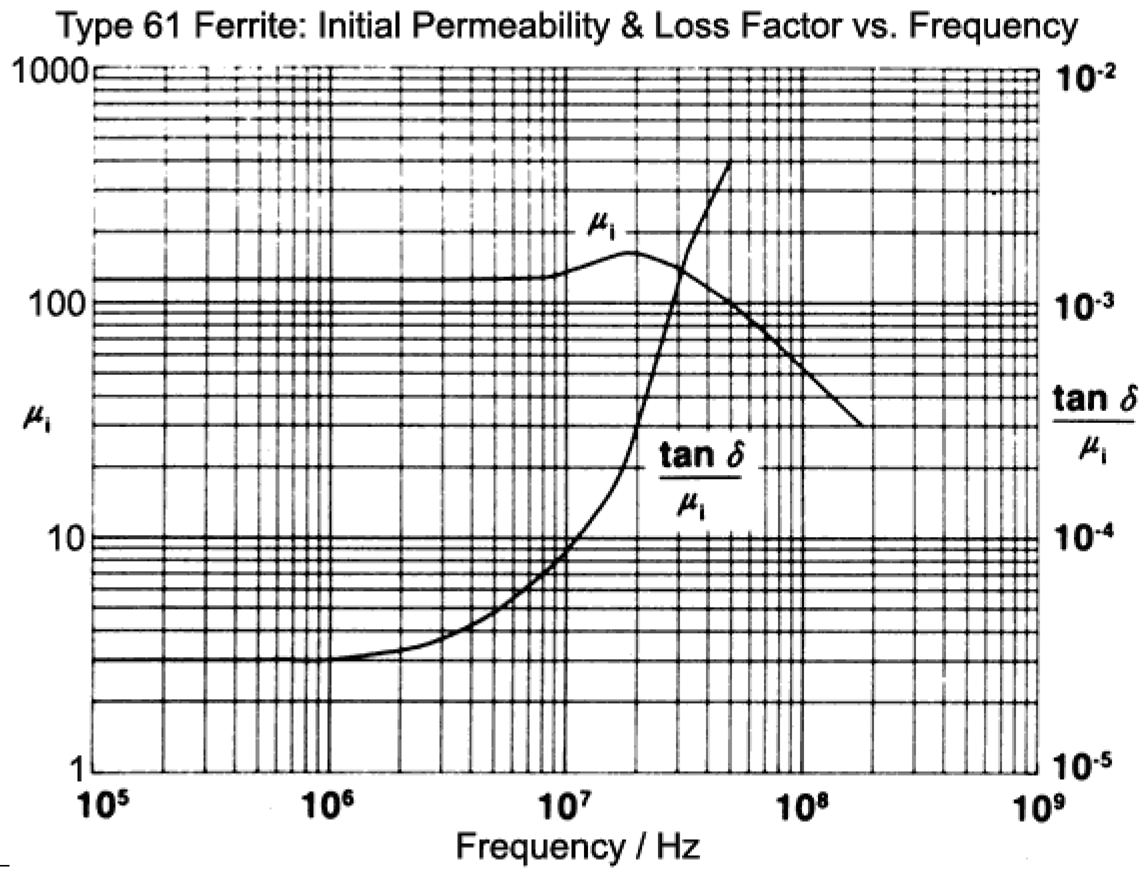
In a separate article[7], a formula relating the transformer parallel loss-resistance to the transfer efficiency factor was given as:

$R_k = k' R_i / (1 - k')$

where R_i is the load on the transformer during the efficiency measurement, and k' is the factor by which the output voltage falls short of that of an ideal transformer. It was found that for transformers wound on type FT50-61 toroids, with 8 to 12 turns on the secondary and a 1-turn primary, k' was about 0.965 ±0.01 at 30 MHz when R_i was 50 Ω.

For the purpose of designing ordinary transmission bridges, it is sufficient to assume that k' does not depend on frequency and adopt a value equivalent to the worst case. For the maximally-flat bridge however, R_{jk} is a low-frequency balance-tracking parameter (albeit a minor one) and so we need to adopt a k' value that is appropriate for the region in which the high-pass network does its work. This, presuming that we are designing with the HF spectrum in mind, is somewhere in the region from 2 MHz to 4 MHz.

Shown below is the graph of complex permeability for type 61 ferrite:



7 **Current transformer efficiency factor** (DWK)

From the graph we can see that the material is entering a dispersion region on moving upwards through the HF spectrum; which means that we can expect greater efficiency at 3 MHz than at 30 MHz. It is extremely difficult to convert this information into an accurate value for k', but an educated guess that will not be far from the mark says that k' at 3 MHz will be somewhere between 0.98 and 0.99. It is reasonable therefore to model the maximally-flat bridge on the basis that $k' = 0.985 \pm 0.005$, an assumption that has the virtue of placing the upper edge of the 99.7% confidence interval at 1. Taking $k' = 0.985 \pm 0.005$, we get:

$$R_k = 50\,k' / (1 - k') = 3283\ (+1667, -833)\ \Omega$$

or, taking the average of the asymmetric uncertainty:

$$R_k = 3283 \pm 1250\ \Omega$$

This estimate is, of course, crude; but there are various reasons for supposing that the uncertainty will not matter. Firstly, in the spreadsheet calculation discussed above, it was found that there was very little difference between the -3 dB point and the phase crossover frequency. This means that R_{jk} has only a small effect on the frequency response. Secondly, R_k is a lot larger than the various calculated values for R_{jk} (600 Ω to 800 Ω for viable designs); which means that R_j will be only a little greater than R_{jk}. The overall uncertainty of an asymmetric parallel combination is weighted towards the uncertainty in the lowest-value component. Hence, it appears doubtful that there will be any need to adjust R_j on test. For those who have the facility to measure the impedance of the secondary winding in the 1.6 MHz to 4 MHz region moreover; it is possible to obtain a fair estimate for R_k directly and so refine the value for R_j.

Although the estimation procedure given above is rough, it nevertheless allows us to split the transformer load into its three resistive components R_h, R_j and R_k. This puts a value on the resistor R_j; and also, if we care to specify the maximum power to be transmitted through the bridge, allows us to calculate the resistor power ratings.

When calculating resistive losses, it is sensible to do so in the worst case. This occurs when all of the reactances have vanished from the system, i.e., in the high-frequency limit of the prototype model when $X_{Li} \rightarrow \infty$ and $X_{Ch} \rightarrow 0$. We will start by allocating the symbol P_0 to the maximum transmitted power; i.e., P_0 is the power dissipated in the main load resistance R_0 when the generator is operating at the design maximum output level and the load is purely resistive. We obtain the voltage appearing across R_0 thus:

$$V = \sqrt{(P_0 R_0)}$$

It is often convenient to adopt $P_0 = 100$ W, firstly because this is a reasonable design criterion for high-sensitivity transmission bridges, and secondly, because all of the power losses calculated from it are then in %. When $P_0 = 100$ W and $R_0 = 50\ \Omega$, $V = 70.7$ V RMS.

From equation **(1.1.1)**, the voltage across the current transformer secondary winding when all reactances have disappeared is:

$$V_i = V\,R_{ik} / (N\,R_0)$$

Hence the maximum power dissipated in (say) R_h is:

$$P_h = V_i^2 / R_h$$

i.e.:

$P_h = P_0 R_{ik}^2 / (N^2 R_0 R_h)$

and so on.

On the basis of the discussion above, the spreadsheet calculation can be extended as follows:

Additional input parameters:	k' (or R_k) , P_0
Formulae:	See **maxflat_prelim.ods** for actual implementation
$R_k = 50 k' / (1 - k')$	Nominal core loss resistance.
$R_j = R_k R_{jk} / (R_k - R_{jk})$	Secondary direct shunt resistance
$P_k = P_0 R_{ik}^2 / (N^2 R_0 R_k)$	Nominal core loss
$P_j = P_0 R_{ik}^2 / (N^2 R_0 R_j)$	Power in R_j
$P_h = P_0 R_{ik}^2 / (N^2 R_0 R_h)$	Power in R_h
$P_{ik} = P_h + P_j + P_k$	Total power in current sampling network.

1.9 Trial voltage-sampling networks

The voltage-sampling network parameters are essentially scaled versions of the current-network parameters, but there is a compromise involved in the choice of the scaling ratio. The problem is that if we make the upper voltage-sampling resistance R_2 too small, the power dissipation in the network will be excessive; but if we make it too large, the compensation inductance L_v will become impractically large and the stray capacitance across R_2 will make a significant contribution to the upper arm impedance.

When all of the reactances in the system disappear, the voltage across the voltage-sampling network is:

$$V' = V + V_{ii} = V \left[1 + R_{ik} / (R_0 N^2) \right]$$

but

$$V = \sqrt{(P_0 R_0)}$$

hence

$$V' = [1 + R_{ik} / (R_0 N^2)] \sqrt{(P_0 R_0)}$$

The voltage across R_2 is:

$$V_2 = V' R_2 / (R_1 + R_2)$$

i.e.,

$$V_2 = [1 + R_{ik} / (R_0 N^2)] [\sqrt{(P_0 R_0)}] R_2 / (R_1 + R_2)$$

The power in R_2 is:

$$P_2 = V_2^2 / R_2$$

i.e.,

$$P_2 = P_0 R_0 [1 + R_{ik} / (R_0 N^2)]^2 R_2 / (R_1 + R_2)^2$$

Similarly:

$$P_1 = P_0 R_0 [1 + R_{ik} / (R_0 N^2)]^2 R_1 / (R_1 + R_2)^2$$

Thus the calculations can be extended to evaluate voltage-sampling networks:

Additional input parameter: R_2	Equation
Formulae: (See **maxflat_brgd.ods** , sheet 1, for implementation).	
$R_1 = R_h R_2 / (N R_0)$	**1.3.10**
$P_1 = P_0 R_0 [1 + R_{ik} / (R_0 N^2)]^2 R_1 / (R_1 + R_2)^2$	
$P_2 = P_0 R_0 [1 + R_{ik} / (R_0 N^2)]^2 R_2 / (R_1 + R_2)^2$	
$P_{1+2} = P_1 + P_2$	
$L_v = L_i R_1 / R_h$	**1.3.10**
$C_v = C_h R_h / R_1$	**1.3.10**

In the spreadsheet **maxflat_brgd.ods**, the row for N = 12 has been singled out for special attention. The value of R_{ik} has been adjusted to make $R_0 = 50\ \Omega$ for that case. The A_L value has also been reduced to make the 1% drop-off point occur at 1.6 MHz; producing the information that the < 1% LF drop-off criterion can be met by using a core with $A_L \geq 63$ nH .

It was found that, with $R_2 = 2.2\ \text{k}\Omega$, the maximum power dissipated in the voltage sampling network will be about 2% of the transmitted power. The corresponding value for L_v is 33 µH. This is a large inductance, but the drawback is not so great as in the case of the current transformer. The point is that a large transformer propagation-delay (which manifests itself as self-capacitance) requires invasive neutralisation arrangements; whereas the parallel capacitance of the lower voltage-sampling network can be balanced-out by placing capacitance across the upper voltage-sampling arm. Indeed, there will be situations in which it will be necessary to place additional capacitance across the lower network in order to offset the strays across the upper network (see section 3).

As shown in the spreadsheet, if the magnetic core used for the voltage sampling network inductor is identical to the one used in the current transformer, then the required inductance can be obtained using 23 turns. In practice, the A_L values of the two cores will not be exactly the same, and it might be necessary to adjust the turns number accordingly. It is also necessary to adjust the inductance fairly exactly in order to obtain perfect frequency-response tracking; for which reason it is advisable to use about 1 turn less than the nearest integer value and place a small adjustable coil in series to make-up the difference.

Note that, even if the two magnetic cores are not nominally identical, it is still advisable to use the same material in both. The reason is that magnetic materials, particularly ferrites of reasonably high permeability, have a large temperature coefficient of permeability. This translates into a large temperature coefficient of inductance; but if the two cores have the same proportionate temperature coefficient, which they will if the material is the same in both, then the frequency tracking will hold as the temperature varies above and below its value at the time of calibration. A series adjustment coil, incidentally, will not make much difference to the temperature tracking as long as its inductance is a small part of the total.

1.10 Multi-turn primaries

As was mentioned during the derivation of equation **(1.5.1)**, the expression for the drop-off criterion f_η must be multiplied by $1/N_p^2$ if the current-transformer has more than one turn on the primary. In that case also, the secondary inductance is given by:

$L_i = A_L N_s^2$

and

$N = N_s / N_p$

For the sake of generality, the ability to evaluate bridges that have an arbitrary number of primary turns has been included in the spreadsheet **maxflat_mtpri.ods**. Note that a bridge with a multi-turn primary will have a high insertion impedance and must either have a very large L_v or dissipate a large percentage of the transmitted power in the voltage sampling network. Such bridges however, have high detector sensitivity and so can be used with low-power generators.

Additional input parameter: N_p	Equation
Formulae: (See **maxflat_mtpri.ods** for implementation).	
$N = N_s / N_p$	
$L_i = A_L N_s^2$	
$U = N^2 R_0 / [\ (N-1)\ R_{ik}\ ]$	**1.5.3**
$f_\eta = R_0 / [\ (\surd 2)\pi\ A_L N_p^2\ (N-1)\ \surd\{\ -1 + \surd[\ 1 + [(1/\eta_f^2)-1\ ]\ 16\ U^2\ (U-1)^2\ ]\ \}\ ]$	(**1.5.2**) / N_p^2

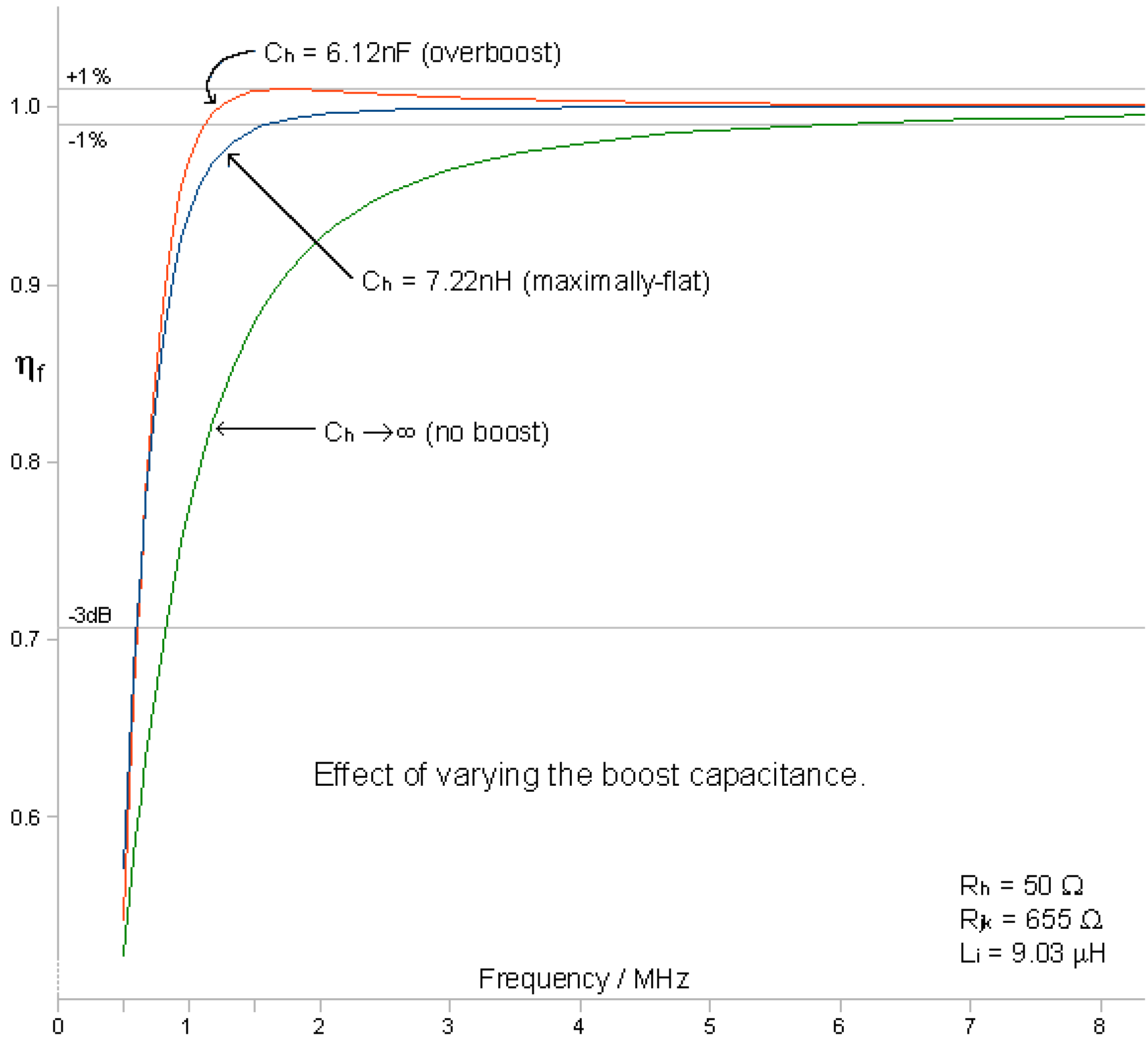
1.11 Overboost

The detector amplitude vs. frequency response for the maximally-flat bridge can be plotted using equation (**1.4.5**). It is rather more interesting however, to plot a version of the response function in which C_h is allowed to vary independently. This allows us to explore the effect of component tolerances, and to see whether there is any advantage in deviating from the exact maximally-flat condition.

The required response function is given by dividing equation (**1.4.4**) by equation (**1.4.2**):

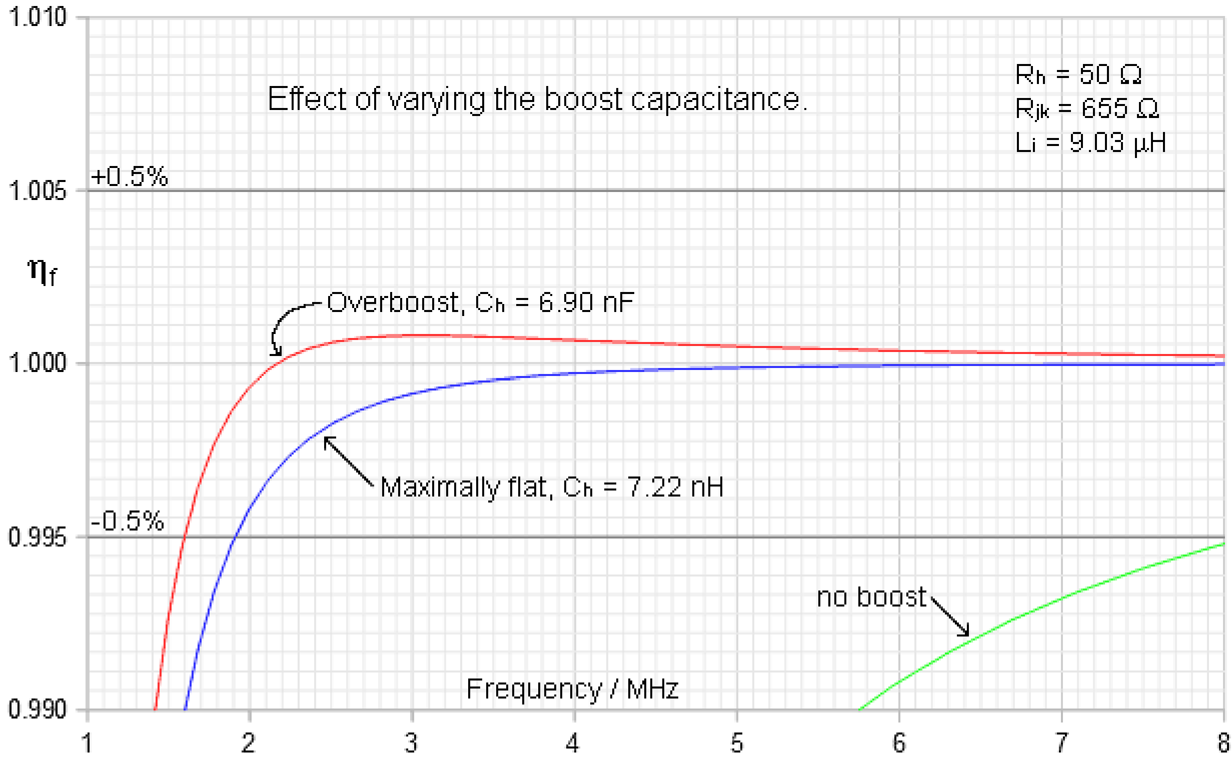
$$\eta_f = (1+R_h/R_{jk}) \Big/ \sqrt{\left[(1+R_h/R_{jk})^2 + \frac{X_{Ch}^2}{X_{Li}^2} + \frac{X_{Ch}^2}{R_{jk}^2} + \frac{2X_{Ch}}{X_{Li}} + \frac{R_h^2}{X_{Li}^2} \right]} \qquad \textbf{(1.11.1)}$$

This function is shown plotted below for a candidate bridge that has been adjusted to have its 1% drop-off point at 1.6 MHz when the maximally-flat condition is imposed (see: **maxflat_brgd.ods** , sheet 2).



The graph shows that when the boost capacitance is reduced below the value required for maximal flatness, an overboost occurs. Specifically, for the case examined, reducing C_h by 15% moves the -1% point to 1.1 MHz, and gives a response of +1% at 1.7 MHz. Broadly, this tells us that there is considerable latitude in the choice of C_h, and it is best to err on the low side. Whatever the choice of C_h however, C_v must always be kept in the correct proportion to it in order to maintain the frequency-independence of the balance condition.

In the graph below, a small amount of overboost has allowed the drop-off to be kept within 0.5% at 1.6 MHz. This particular adjustment has also kept the maximum boost (which occurs in the region of 3 MHz) to about 0.15 % (see: **maxflat_brgd.ods** , sheet 3)

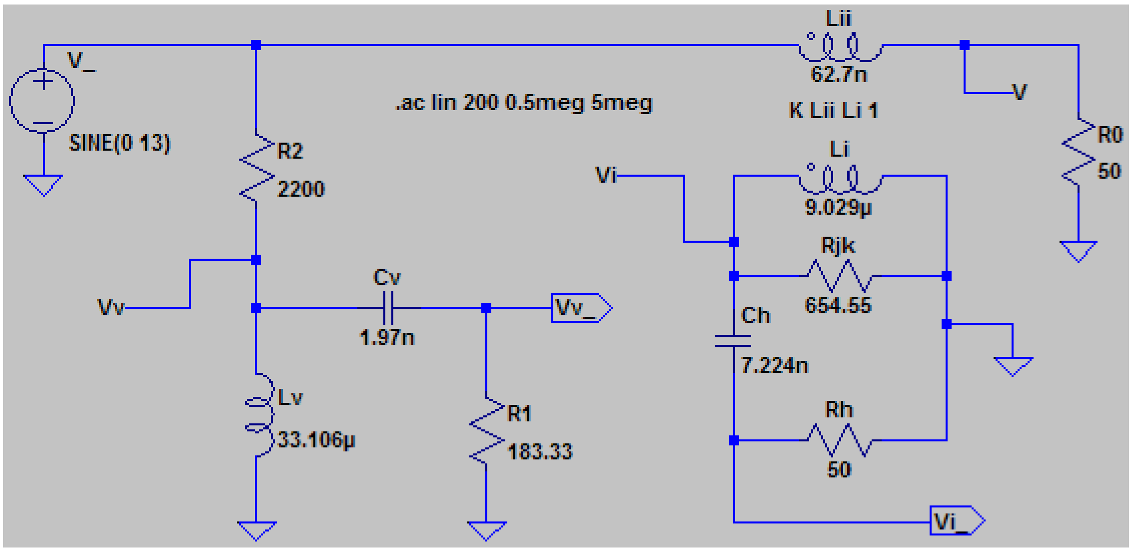


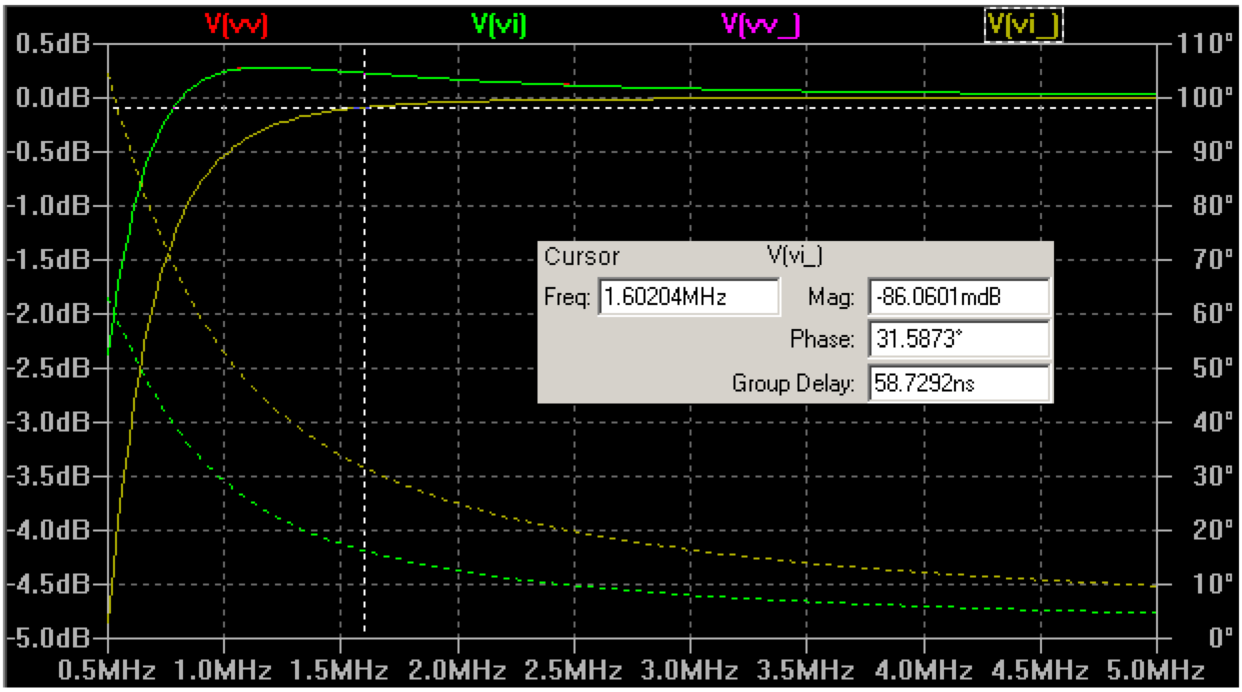
It is evident that, by judicious adjustment of circuit parameters, it would be possible to create networks that are flat within ±0.1% over many octaves.

2 SPICE simulations

a) Individual sampling networks

The circuit file **maxflat_v_i.asc** is used to simulate the voltages appearing on the voltage and current sampling networks. It can be opened using the free simulator LTspice[8]. The schematic is shown below. The circuit is not wired as a bridge because the intention in this case is to plot the voltages separately.





The simulation confirms the theory developed in the preceding sections. With the through-line terminated in the target load resistance R_0, V_v is identical to V_i, and the two outputs V_v' and V_i' are identical (an underscore is used instead of a prime because apostrophes are not allowed in netlists). The cursor is shown placed on the curve for V_i' at 1.6 MHz. The dB reading (obtained by dividing the mdB value by 1000) can be converted into relative output thus:

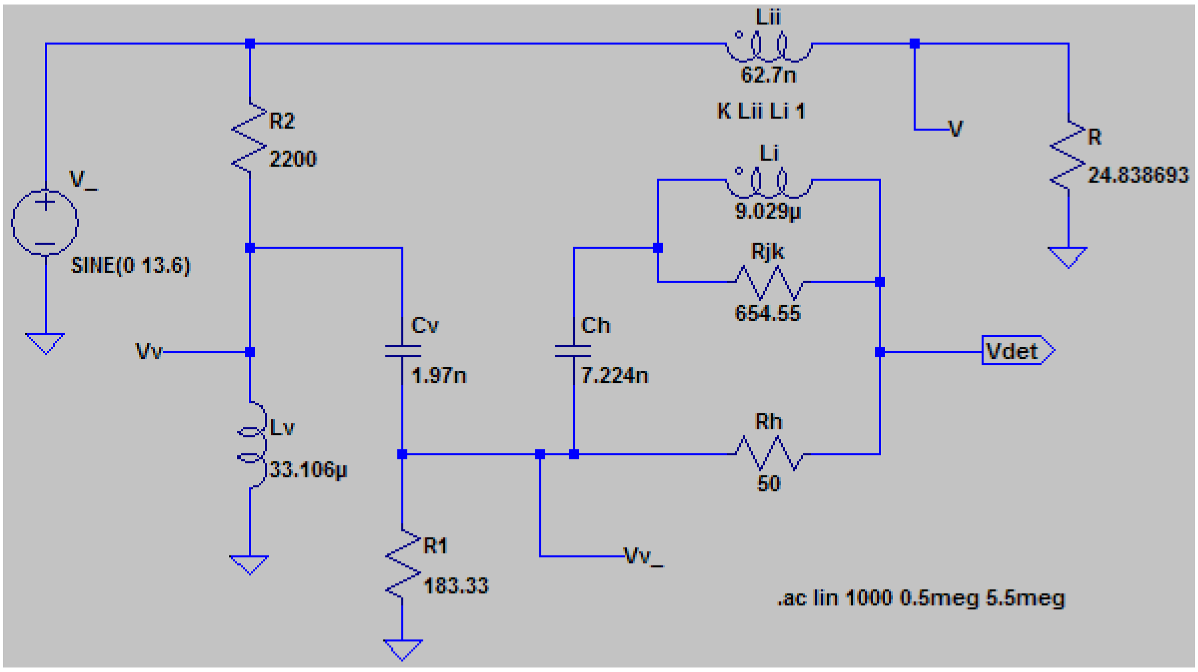
8 http://www.linear.com/designtools/software/

$10^{-(0.08606\,/\,20)} = 0.99$ [volts per volt]

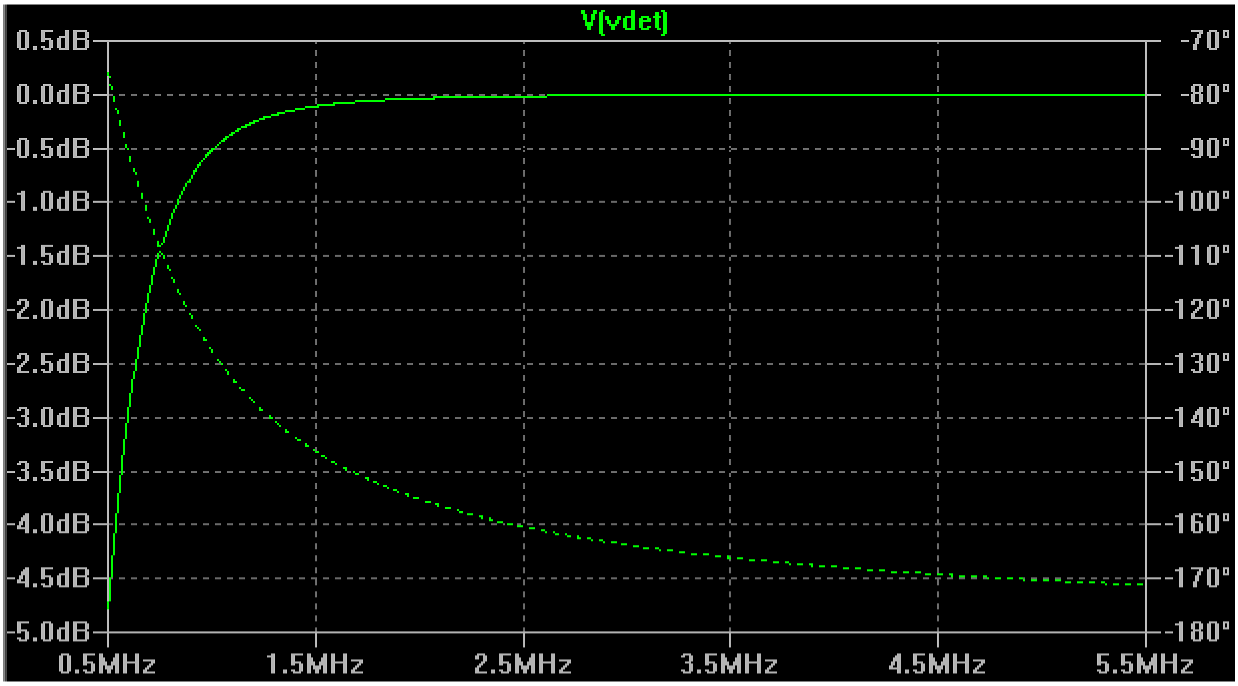
Note that there is a hump in V_v (and V_i) due to the falling reactance of the boost capacitor as the frequency is reduced.

b) Bridge-connected sampling networks

The circuit file **maxflat_brg.asc** has the sampling networks connected in series opposition (i.e., as a bridge). This configuration gives no output when the through-line is terminated in its target load resistance (50 Ω), and so the frequency response simulation is carried out with a mismatched load. The load value of 24.838693 Ω shown in the schematic below has no significance except that it gives a detector output of 0 dB at 30 MHz. This gives an easy confirmation of the point proved in section **1.4**; which is that the line terminating impedance makes no difference to the relative frequency responses of the individual sampling networks, and hence makes no difference to the relative frequency response of the output obtained from the detector port.



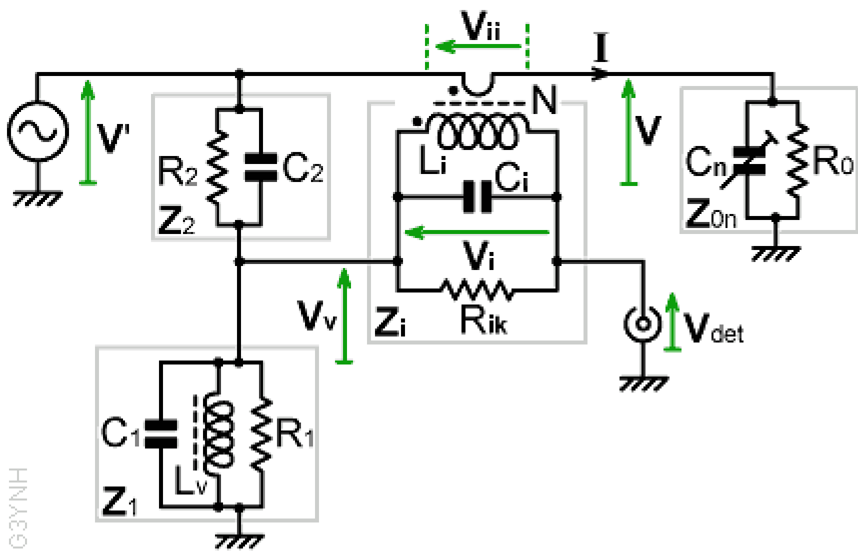
The frequency response shown in the graph below is proportionately identical (i.e., identical in overall shape) to that of the individual sampling networks. Changing the main load value (R) changes the output level, but not the shape of the response curve.


V(vdet)
0.5dB
0.0dB
-0.5dB
-1.0dB
-1.5dB
-2.0dB
-2.5dB
-3.0dB
-3.5dB
-4.0dB
-4.5dB
-5.0dB
-70°
-80°
-90°
-100°
-110°
-120°
-130°
-140°
-150°
-160°
-170°
-180°
0.5MHz
1.5MHz
2.5MHz
3.5MHz
4.5MHz
5.5MHz

3. High-frequency RVS model

To make a bridge that will balance correctly over the entire 4¼-octave short-wave spectrum, it is always necessary to include some kind of high-frequency compensation. In particular; we must do something about the effective secondary parallel capacitance of the current-transformer, and we must take the parasitic capacitances of the upper and lower voltage-sampling arms into account. All of these capacitances will have a significant effect at high frequencies and so must be put into the model; but the analytical problem can be simplified by noting that they are all of the order of a few pF. It follows that any low-frequency response-tailoring scheme (involving capacitances of several nF) will only be affected at about the 0.1% level by such tiny capacitances. Hence we can devise our HF neutralisation arrangements by reference to the conventional resistive voltage-sampling (RVS) bridge; i.e., we can short-out any boost capacitors and forget them during this part of the analysis.

Shown below is an equivalent circuit for the RVS bridge with parasitic capacitances included. The presence of a Faraday-shield is implied by the absence of a stray capacitance from the through-line to the detector port. The circuit includes a neutralising capacitor C_n placed in parallel with the load resistance. This capacitor should be considered to represent a generic current-transformer HF phase-neutralisation scheme; i.e., neutralisation can be accomplished in various ways[9] but, in terms of their effect on the balance condition, all such techniques are equivalent to the inclusion of C_n.



RVS bridge with parasitic capacitances and neutralising capacitor (C_n).

C_n is included in advance of any analysis because it is obvious by inspection that there can be no frequency-independent solutions for the balance condition in the absence of a neutralisation network. This point can be understood by considering the four principal impedances shown in grey boxes. These, subject to transformation in the case of the primary load impedance, are analogous to the four impedance-arms of a Wheatstone-Christie bridge. The voltage-sampling network consists of an RLC network and an RC network. The current-transformer is an RLC network, and so the primary load must behave as an RC network if all of the frequency factors are to drop-out of the balance relationships.

9 **Evaluation and optimisation of current transformer bridges** (DWK), section 18.

In order to find the balance conditions, we can use much the same procedure as was employed in section **1.3**. This involves writing expressions for $\mathbf{V}_i$ and $\mathbf{V}_v$ (both derived from the same reference voltage, $\mathbf{V}$) and equating them when the through-line is terminated in the target load resistance R_0 . The resulting expression is then rearranged to get all of the parallel impedances into reciprocal form, so that each can be expanded into a series of admittances.

In this case, our generic neutralisation method places a capacitance (actual or virtual) in parallel with the load, so that the total load impedance at balance is complex, i.e.:

$$\mathbf{Z}_{0n} = R_0 // \mathbf{j}X_{Cn}$$

Hence, by the ampere-turns rule and because $\mathbf{I} = \mathbf{V} / \mathbf{Z}_{0n}$, the output of the current transformer is:

$$\mathbf{V}_i = \mathbf{V}\,\mathbf{Z}_i / (\,\mathbf{Z}_{0n}\,N\,)$$

where

$$\mathbf{Z}_i = R_{ik} // \mathbf{j}X_{Li} // \mathbf{j}X_{Ci}$$

The output of the voltage-sampling potential-divider is:

$$\mathbf{V}_v = \mathbf{V}'\,\mathbf{Z}_1 / (\,\mathbf{Z}_1 + \mathbf{Z}_2\,)$$

$$= \mathbf{V}'\,(\,\mathbf{Z}_1 // \mathbf{Z}_2\,) / \mathbf{Z}_2$$

where

$$\mathbf{V}' = \mathbf{V} + \mathbf{V}_{ii} = \mathbf{V}\,[\,1 + \mathbf{Z}_i / (\,\mathbf{Z}_{0n}\,N^2\,)\,]$$

$$\mathbf{Z}_1 = R_1 // \mathbf{j}X_{Lv} // \mathbf{j}X_{C1}$$

and

$$\mathbf{Z}_2 = R_2 // \mathbf{j}X_{C2}$$

To balance the bridge, we set $\mathbf{V}_v = \mathbf{V}_i$ and cancel $\mathbf{V}$. Thus, in compact form:

$$[\,1 + \mathbf{Z}_i /(\,\mathbf{Z}_{0n}\,N^2\,)\,]\,(\,\mathbf{Z}_1 // \mathbf{Z}_2\,) / \mathbf{Z}_2 = \mathbf{Z}_i / (\,\mathbf{Z}_{0n}\,N\,)$$

Now, taking the reciprocal, and moving $[\,1+\mathbf{Z}_i /(\,\mathbf{Z}_{0n}\,N^2\,)]$ to the right-hand side:

$$\mathbf{Z}_2 / (\mathbf{Z}_1 // \mathbf{Z}_2\,) = [\,1 + \mathbf{Z}i / (\,\mathbf{Z}_{0n}\,N^2\,)]\,(\,\mathbf{Z}_{0n}\,N\,) / \mathbf{Z}_i$$

Multiplying-out the right-hand side gives:

$$\mathbf{Z}_2 / (\,\mathbf{Z}_1 // \mathbf{Z}_2\,) = (\,\mathbf{Z}_{0n}\,N / \mathbf{Z}_i\,) + 1/N$$

Expanding $1 / (\,\mathbf{Z}_1 // \mathbf{Z}_2\,)$ and $1 / \mathbf{Z}_i$ gives:

$$\mathbf{Z_2}\left[\frac{1}{\mathbf{Z_1}}+\frac{1}{\mathbf{Z_2}}\right]=\mathrm{N}\,\mathbf{Z_{0n}}\left[\frac{1}{R_{ik}}+\frac{1}{\mathbf{j}X_{Li}}+\frac{1}{\mathbf{j}X_{Ci}}\right]+\frac{1}{N}$$

Multiplying $\mathbf{Z_2}$ into the bracket gives the left hand side as $(\mathbf{Z_2}/\mathbf{Z_1})+1$. Then subtracting 1 from both sides gives:

$$\frac{\mathbf{Z_2}}{\mathbf{Z_1}}=\mathrm{N}\,\mathbf{Z_{0n}}\left[\frac{1}{R_{ik}}+\frac{1}{\mathbf{j}X_{Li}}+\frac{1}{\mathbf{j}X_{Ci}}\right]+\frac{1}{N}-1$$

We now need to expand $\mathbf{Z_1}$ and $\mathbf{Z_2}$. This is best accomplished by moving $\mathbf{Z_2}$ to the right-hand side, where it becomes an admittance. Also a certain proliferation of brackets is prevented by multiplying $\mathrm{N}\,\mathbf{Z_{0n}}$ into the bracket on the right-hand side before we do so:

$$\frac{1}{\mathbf{Z_1}}=\frac{1}{\mathbf{Z_2}}\left[\frac{\mathrm{N}\,\mathbf{Z_{0n}}}{R_{ik}}+\frac{\mathrm{N}\,\mathbf{Z_{0n}}}{\mathbf{j}XLi}+\frac{\mathrm{N}\,\mathbf{Z_{0n}}}{\mathbf{j}X_{Ci}}+\frac{1}{N}-1\right]$$

Expanding $\mathbf{Z_1}$ and $\mathbf{Z_2}$ gives:

$$\frac{1}{R_1}+\frac{1}{\mathbf{j}X_{Lv}}+\frac{1}{\mathbf{j}X_{C1}}=\left[\frac{1}{R_2}+\frac{1}{\mathbf{j}X_{C2}}\right]\left[\frac{\mathrm{N}\,\mathbf{Z_{0n}}}{R_{ik}}+\frac{\mathrm{N}\,\mathbf{Z_{0n}}}{\mathbf{j}X_{Li}}+\frac{\mathrm{N}\,\mathbf{Z_{0n}}}{\mathbf{j}X_{Ci}}+\frac{1}{N}-1\right]$$

And multiplying-out the brackets [noting that $(1/N)-1=-(N-1)/N$ and that $\mathbf{j}^2=-1$]:

$$\frac{1}{R_1}+\frac{1}{\mathbf{j}X_{Lv}}+\frac{1}{\mathbf{j}X_{C1}}=\frac{\mathrm{N}\,\mathbf{Z_{0n}}}{R_{ik}R_2}+\frac{\mathrm{N}\,\mathbf{Z_{0n}}}{\mathbf{j}X_{Li}R_2}+\frac{\mathrm{N}\,\mathbf{Z_{0n}}}{\mathbf{j}X_{Ci}R_2}-\frac{N-1}{N\,R_2}+\frac{\mathrm{N}\,\mathbf{Z_{0n}}}{\mathbf{j}X_{C2}R_{ik}}-\frac{\mathrm{N}\,\mathbf{Z}0n}{X_{Li}X_{C2}}-\frac{\mathrm{N}\,\mathbf{Z_{0n}}}{X_{Ci}X_{C2}}-\frac{N-1}{\mathbf{j}N\,X_{C2}}$$

The full expansion requires the substitution $\mathbf{Z_{0n}}=R_0 // \mathbf{j}X_{Cn}$. This is best accomplished by moving all of the terms with $\mathbf{Z_{0n}}$ as a factor to one side of the equation, then dividing both sides by $1/\mathbf{Z_{0n}}$. Thus (also recalling that $X_L X_C=-L/C$):

$$\left[\frac{1}{R_1}+\frac{1}{\mathbf{j}X_{Lv}}+\frac{1}{\mathbf{j}X_{C1}}+\frac{N-1}{N\,R_2}+\frac{N-1}{\mathbf{j}N\,X_{C2}}\right]\left[\frac{1}{R_0}+\frac{1}{\mathbf{j}X_{Cn}}\right]$$

$$=\frac{N}{R_{ik}R_2}+\frac{N}{\mathbf{j}X_{Li}R_2}+\frac{N}{\mathbf{j}X_{Ci}R_2}+\frac{N}{\mathbf{j}X_{C2}R_{ik}}+\frac{N\,C_2}{L_i}-\frac{N}{X_{Ci}X_{C2}}$$

Multiplying-out the left-hand side gives the final expansion, where all terms have dimensions of [1/Ω²]:

$$\frac{1}{R_0 R_1} + \frac{1}{jR_0 X_{Lv}} + \frac{1}{jR_0 X_{C1}} + \frac{N-1}{N R_0 R_2} + \frac{N-1}{jN R_0 X_{C2}} + \frac{1}{jR_1 X_{Cn}} + \frac{C_n}{L_v} - \frac{1}{X_{C1} X_{Cn}} + \frac{N-1}{jN R_2 X_{Cn}} - \frac{N-1}{N X_{C2} X_{Cn}}$$

$$= \frac{N}{R_{ik} R_2} + \frac{N}{jX_{Li} R_2} + \frac{N}{jX_{Ci} R_2} + \frac{N}{jX_{C2} R_{ik}} + \frac{N C_2}{L_i} - \frac{N}{X_{Ci} X_{C2}} \qquad (3.1)$$

The real part of this expression corresponds to the in-phase balance condition, and the imaginary part corresponds to the quadrature balance condition. Equating the reals gives:

$$\frac{1}{R_0 R_1} + \frac{N-1}{N R_0 R_2} + \frac{C_n}{L_v} - \frac{1}{X_{C1} X_{Cn}} - \frac{N-1}{N X_{C2} X_{Cn}} = \frac{N}{R_{ik} R_2} + \frac{N C_2}{L_i} - \frac{N}{X_{Ci} X_{C2}} \qquad (3.2)$$

From this, we can see that frequency-independence of the in-phase balance conditon is obtained when:

$$\frac{1}{X_{C1} X_{Cn}} + \frac{N-1}{N X_{C2} X_{Cn}} - \frac{N}{X_{Ci} X_{C2}} = 0$$

i.e., using $X_C = -1/(2\pi f\, C)$:

$$C_1 C_n + \frac{(N-1)\, C_2 C_n}{N} - N\, C_i C_2 = 0$$

Dividing throughout by C_2 gives:

$$C_n \left[\frac{C_1}{C_2} + 1 - \frac{1}{N} \right] = N\, C_i \qquad (3.3)$$

Equating the imaginaries in **(3.1)** gives:

$$\frac{1}{R_0 X_{Lv}} + \frac{1}{R_0 X_{C1}} + \frac{N-1}{N R_0 X_{C2}} + \frac{1}{R_1 X_{Cn}} + \frac{N-1}{N R_2 X_{Cn}} = \frac{N}{X_{Li} R_2} + \frac{N}{X_{Ci} R_2} + \frac{N}{X_{C2} R_{ik}} \qquad (3.4)$$

But from the requirements for low-frequency balance given earlier as equation (**1.3.10**):

$L_v = L_i R_2 / (N R_0)$

Hence the terms $1/(R_0 X_{Lv})$ and $N/(X_{Li} R_2)$ cancel (for which reason they are in grey above). This leaves only terms involving capacitive susceptance, and so using $X_C = -1/(2\pi f\, C)$ and cancelling $-2\pi f$ throughout:

$$\frac{C_1}{R_0} + \frac{(N\text{-}1)\,C_2}{N\,R_0} + \frac{C_n}{R_1} + \frac{(N\text{-}1)\,C_n}{N\,R_2} = \frac{N\,C_i}{R_2} + \frac{N\,C_2}{R_{ik}}$$

Muttiplying throughout by R_2 and regrouping gives:

$$\frac{C_1 R_2}{R_0} + C_2 R_2 \left[\frac{(N\text{-}1)}{N\,R_0} - \frac{N}{R_{ik}} \right] + C_n \left[\frac{R_2}{R_1} + 1 - \frac{1}{N} \right] = N\,C_i \qquad (\mathbf{3.5})$$

Although, on the diagram above, only the neutralisation capacitor is marked as adjustable, any of the stray capacitances can be padded to a higher value if necessary. Hence, in **(3.3)** and **(3.5)** it appears that we have two simultaneous equations with four adjustable parameters (C_1, C_2, C_i, C_n). This means that there are either an infinite number of ways in which high-frequency balance tracking can be accomplished; *or*, we are missing some crucial piece of information. The latter is, of course, the case; and in deducing the solution we eliminate the paradox. In the limit where X_{Lv} is very large (i.e. at high frequencies), the voltage-sampling network should be considered as a resistive potential-divider in parallel with a capacitive potential-divider. It is possible to make separate resistive and capacitive dividers that give exactly the same off-load output voltage, the only difference being the output impedances. If the outputs of such a pair of networks are connected together, there will be no changes in the output voltages. If the division ratios of the two networks are not the same however, then the combined output voltage will vary with frequency. Thus, in order to obtain a flat high-frequency response, we must impose the condition:

$R_2 / R_1 = X_{C2} / X_{C1}$

i.e.:

$R_2 / R_1 = C_1 / C_2$	**3.6**

Substituting this into **(3.3)** gives:

$$C_n \left[\frac{R_2}{R_1} + 1 - \frac{1}{N} \right] = N\,C_i \qquad (\mathbf{3.7})$$

A check of the reasoning used in the derivation above can be had by substituting **(3.7)** into **(3.5)**. The result, after cancellation is:

$$\frac{C_1}{R_0} + C_2 \left[\frac{(N\text{-}1)}{N\,R_0} - \frac{N}{R_{ik}} \right] = 0$$

Multiplying throughout by R_0 and rearranging gives:

$$\frac{C_1}{C_2} = \frac{N R_0}{R_{ik}} + \frac{1}{N} - 1$$

This is the transformer constant, introduced earlier as equation **(1.3.12)**. Hence we can re-write **(1.3.12)** with this supplementary information:

$$\frac{C_1}{C_2} = \frac{R_2}{R_1} = \frac{N R_0}{R_{ik}} + \frac{1}{N} - 1 \qquad \text{Transformer constant} \qquad (3.8)$$

This result confirms the deduction (**3.6**) and the logical consistency of the working. Also, we can use it to substitute for R_2 / R_1 in equation (**3.7**) to give:

$C_i / C_n = R_0 / R_{ik}$	**3.9**

Now returning to equation **(3.2)**; notice that when the high-frequency balance condition **(3.3)** is applied, equation **(3.2)** reduces to:

$$\frac{1}{R_0 R_1} + \frac{N-1}{N R_0 R_2} + \frac{C_n}{L_v} = \frac{N}{R_{ik} R_2} + \frac{N C_2}{L_i}$$

Multiply throughout by $R_0 R_2$ and rearranging gives:

$$\frac{R_2}{R_1} + 1 - \frac{1}{N} - \frac{N R_0}{R_{ik}} + \frac{R_0 R_2 C_n}{L_v} = \frac{N R_0 R_2 C_2}{L_i}$$

which, applying the cancellation given by equation **(3.8)** and then dividing throughout by $R_0 R_2$ leaves us with:

$C_n / L_v = N C_2 / L_i$

This is an auxiliary balance condition that links the frequency response of the voltage-sampling network to that of the current-sampling network. It supplements the collected balance relationships given earlier as equation **(1.3.10)**.

$$\frac{R_1}{R_h} = \frac{L_v}{L_i} = \frac{C_h}{C_v} = \frac{R_2}{N R_0} = \frac{C_n}{N C_2} \qquad (3.9)$$

From **(3.9)** we obtain the additional relationship:

$C_n / C_2 = R_2 / R_0$	**3.10**

Hence, rather than being free to choose the various network capacitances, we find that their relative values must be constrained if we are to get the bridge to balance at high frequencies. If only one of the capacitances is determined by practical considerations (such as the desire to keep them all as small as possible) then all of the others are prescribed. This situation is remarkably different from that encountered when designing capacitive voltage-sampling (CVS, Douma) bridges, because the strays across the voltage-sampling network merely contribute to wanted capacitances in that case. It means that the RVS bridge, although conceptually simple in its prototype (low-frequency model) form, is actually the hardest to get right.

The high-frequency balance considerations have been added to the spreadsheet calculation **maxflat_brgd.ods**. A little experimentation with the numbers confirms that the capacitance across the upper voltage-sampling arm (C_2) is always the smallest of the set, and that R_2 must not be too large if the others are to be kept within reasonable bounds. Thus it is sensible to assume that C_2 will be the capacitance of the resistor R_2 (no extra capacitance will be used in this location). If a single resistor is used, then C_2 will be about 0.5 pF. This makes the other capacitances rather large. Using two resistors in series however reduces C_2 to about 0.25 pF. The other capacitances must then be padded or adjusted to their corresponding values. In the spreadsheet, C_i is chosen as the input value and is adjusted to make C_2 come out at 0.25 pF.

Additional input parameter: C_i	Equation
Formulae: (See **maxflat_brgd.ods** for implementation).	
$C_n = C_i R_{ik} / R_0$	**3.9**
$C_2 = C_n R_0 / R_2$	**3.10**
$C_1 = C_2 R_2 / R_1$	**3.8**

In the given example, R_2 was chosen to be 2.2 kΩ to give about 2.1% power loss in the voltage sampling network (see section **1.9**). With the other parameters as before, and $C_2 = 0.25$ pF, the remaining capacitances were:

$C_i = 11.8$ pF , $C_n = 11.0$ pF and $C_1 = 3.0$ pF.

In a current transformer network, the effective secondary capacitance C_i is principally due to propagation delay and through-line mismatch. It can be made considerably less than 11.8 pF by careful layout and construction (ca. 8 pF say) and so the transformer secondary can be padded to the required value with a trimer capacitor. It is also perfectly possible to place the padding across the transformer primary, except that the value required at the primary will be larger than that required at the secondary by a factor of N^2.

The capacitance C_1 will be mainly provided by the self-capacitance of the coil L_v. It might be difficult to get that as low as 3 pF, but a single layer toroidal winding with a gap between the ends will probably meet the requirement. If not, then C_2 will have to be increased slightly, e.g., by placing a pair of short stiff wires in proximity.

Notice, incidentally, that the final circuit leaves us with two parallel RLC networks. If we assume that the generator is a short circuit, then these are: $L_v // (C_1 + C_2)$ and $L_i // (C_i + C_n / N^2)$. The 'resonant' frequencies for the voltage and current sampling networks are the respectively:

$f_{0v} = 1 / [2\pi\sqrt{\{ L_v (C_1 + C_2) \}}$

and

$f_{0i} = 1 / [\ 2\pi\sqrt\{ L_i (C_i + C_n / N^2) \}$

Both of these frequencies are the same when the bridge is neutralised, and so this relationship can be used as a check the calculation. The resonances however, do not affect the frequency response. They are firstly, heavily damped by parallel resistance; and secondly, they merely constitute the point at which the parallel reactive component of the network becomes open circuit.

Discussion

The investigation carried out in the preceding sections demonstrates the feasibility of using the maximally-flat current transformer network as the basis for a precision transmission bridge. A serious drawback however lies in the necessity of using a resistive voltage sampling network. In the passive RVS network, the amount of power abstracted from the generator is necessarily large (several percent of the total output) because high-frequency neutralising arrangements are difficult to implement unless the upper voltage sampling arm resistance is relatively low.

One possible way of avoiding the difficulties associated with RVS presents itself when the networks are used in conjunction with active circuitry. If a power supply is available, then it is a simple matter to use a capacitive potential divider leading to a small broadband linear power amplifier (i.e., a 'video' line-driving amplifier). The output of the amplifier might then drive a passive low-impedance network that tailors the frequency response to that of the current transformer network (i.e., a scaled-down version of the network used in the fully passive case). The ability to control amplifier feedback also introduces the possibility of alternative filter topologies. The initial capacitive voltage sampling network will benefit from inductance balance adjustment[10] if high precision is required, but the power abstracted from the RF generator will be negligible.

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