Current Transformer Efficiency Factor

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Preface

If the secondary network of a current transformer can be represented as a set of impedances in parallel, and the apparent secondary capacitance can be neutralised, then there will exist a frequency-independent solution for the balance condition of a bridge that uses the transformer as a current-sampling element and a potential-divider as the voltage-sampling element. The purely-parallel model can not, of course, offer a rigorous representation of the network, because both leakage inductance and winding loss are strictly in series with the output; but if these quantities are relatively small, the effect of neglecting them can be absorbed into the other parameters. If that is the case, then the shortfall in output that results from losses and incomplete coupling can be modelled by including a resistance in parallel with the secondary load resistance, and by allowing the effective secondary inductance to be slightly less than the measured secondary inductance. This semi-empirical modification of the 'ideal transformer with secondary inductance' model has been used by the author in the design of transmission-line bridges offering magnitude accuracy of better than 0.1% and phase error of < 0.1° over at least 5 octaves[1]. This article gives the experimental justification for the model.

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1 **Evaluation and optimisation of current transformer bridges**. D W Knight. http://g3ynh.info/zdocs/bridges/

1. Introduction

A reasonably good description of a tightly-coupled current transformer is given by using an ideal transformer model with parallel secondary reactance. The relationship between output voltage and input current for that model is given by the expression:

$\mathbf{V_i} = \mathbf{I}\,\mathbf{Z_i} / N$

where N is the turns ratio and $\mathbf{Z_i}$ is the parallel combination of the secondary load resistance and the secondary reactance, i.e.:

$\mathbf{Z_i} = (R_i \,//\, \mathbf{j}X_{Li} \,//\, \mathbf{j}X_{Ci})$

A potential problem here however is that various transformer non-idealities, particularly core loss, winding resistance and leakage inductance, have been partially neglected. This means that the actual output voltage of the transformer is always slightly less than that predicted by the model. The discrepancy is often of little consequence because it is small in engineering terms; but it is necessary to keep track of it, and if possible to quantify it, when calculating the balance condition for a bridge, or when calibrating an accurate power meter or ammeter.

One way of accounting for the output shortfall is to include an empirical frequency-independent transfer efficiency factor in the input-output relationship. The justification for dealing with the problem in this way is that the *relative* frequency response of the transformer is well described when only parallel reactance is included [2] ; the reason being that, although the losses and inductance leakages are frequency dependent, the dependence is weak in comparison to the major non-idealities and so can be absorbed into the parallel reactance parameters. Hence we might modify the transfer relationship in one of two ways:

Either $\mathbf{V_i} = k'\,\mathbf{I}\,\mathbf{Z_i} / N$ **Or** $\mathbf{V_i} = \mathbf{I}\,(k' R_i \,//\, \mathbf{j}X_{Li} \,//\, \mathbf{j}X_{Ci}) / N$

In the first approach, we simply multiply the predicted output voltage by a factor k' , which is slightly less than one. In the second approach, we multiply the load resistance by a factor k' , in which case we can consider k' to have come about as a result of a parasitic parallel resistance R_k (say) defined such that:

$k' R_i = R_i \,//\, R_k = R_i R_k / (R_i + R_k)$

i.e:

$k' = R_k / (R_i + R_k)$

and by rearrangement:

$R_k = k' R_i / (1 - k')$

Note that the empirical factor k' is not the same as the transformer coupling coefficient k, hence the prime.

There is little difference between the two approaches, except that the first slightly modifies the effective parallel reactances in the process of reducing the output voltage, whereas the latter does not. In other experiments performed by the author, it was found that the value of secondary

2 **Amplitude response of conventional and maximally-flat current transformers**. D W Knight. http://g3ynh.info/zdocs/bridges/

inductance obtained by direct measurement was always slightly greater than the inductance obtained by linear regression analysis of a set of frequency response measurements[3]. Multiplying the measured inductance by a realistic k value brings the two values into agreement, but since the frequency response method gives a very accurate inductance value there seems to be little merit in linking its value to an empirical amplitude correction factor. The difference between directly measured secondary inductance (total inductance) and the value of inductance required to fit the frequency response curve (coupled inductance) is, of course, most closely related to the secondary *leakage inductance* (about 1% of the total). Hence it seems sensible to define L_i as the coupled secondary inductance.

2. Measuring transfer efficiency

Described below is a procedure for measuring the transfer efficiency of current transformers; followed by various measurements made using that procedure. For the purpose of these tests, transfer efficiency is defined as follows:

$$\text{Transfer efficiency} = k' = \frac{|\text{measured output voltage}|}{|\text{predicted output voltage}|}$$

where the measured voltage is a magnitude (a meter reading) and the predicted output voltage is defined as:

$$|\mathbf{V}_{\mathbf{i\,(theoretical)}}| = |\mathbf{I}|\ |(R_i' \,//\, \mathbf{j}X_{Li} \,//\, \mathbf{j}X_{Cis})| \,/\, N$$

and R_i' is the parallel combination of the secondary load resistance and the voltmeter (detector) input resistance.

Notice that the secondary shunt capacitance is given as C_{is} . Only stray capacitance is included, and the 'self-capacitance' of the coil is ignored for the purpose of calculating magnitudes. The reason for that is that none of the transformers tested had more than 20 turns on the secondary, and amplitude frequency-response measurements indicated that all of the transformers were operating in a completely flat region of the frequency response at the measurement frequency of 30 MHz. Had the coil 'self-capacitance' been taken into account, this would have predicted a slight roll-off at 30 MHz, but no such roll-off occurs in practice. The explanation is that 'self-capacitance' is largely fictitious. It is merely a lumped-component representation for the time delay that occurs due to the finite velocity of electromagnetic waves propagating along the winding wire[4]. The transformer secondary is effectively a transmission line, and although unlikely to be terminated exactly in its characteristic impedance, the principal effect of the time delay is to shift the phase of the output without affecting the magnitude. Including a fictitious capacitance in the model would shift the phase of the theoretical output voltage and also reduce its magnitude, inflating the apparent efficiency in some cases to more than 100%. The fact that current transformers do not have gain must advise our choice of model in this case.

The secondary inductance, on the other hand, is a perfectly good lumped parameter. Hence it has been included, even though its effect on the output amplitude at 30 MHz is small. Note that, since the resulting correction is small, the effect of any uncertainty in the measured inductance is negligible.

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3 **Amplitude response of conventional and maximally-flat current transformers** (DWK, Already cited). Also: **Evaluation and optimisation of current transformer bridges**. (DWK, Already cited), **section 3**.

4 **Evaluation and optimisation of current transformer bridges**. as above, **section 16a**.

Using a standard expression for the magnitude of an impedance in parallel form[5] , $|\mathbf{Z_i}|$ can be written:

$$|\mathbf{Z_i}| = R_i' / \sqrt{[1 + (R_i' / X_i)^2]}$$

where:

$$X_i = X_{Li} // X_{Cis}$$

Hence (and also shortening "theoretical" to "theor"):

$$|\mathbf{V_{i(theor)}}| = |\mathbf{I}| R_i' / \{ N \sqrt{[1 + (R_i' / X_i)^2]} \}$$

By expressing the voltage magnitude in this form, the reactive contribution to the output is separable as a correction factor:

$$1/\sqrt{[1 + (R_i' / X_i)^2]}$$

which is intentionally made close to unity by adopting 30 MHz as the measurement frequency, i.e., by choosing a frequency at which the reactances of the secondary inductance L_i and the 'true' parallel capacitance C_{is} approximately cancel.

3. Measurement method

The main experimental problem is that of how to establish an accurately defined RF reference current in the transformer primary and make concomitantly accurate measurements of output voltage and load resistance. Since transformer efficiency is normally very high, systematic errors of a few percent in any of those quantities will make nonsense of the results. The author's solution, using the attenuator, load and diode detectors shown below, was to calibrate the current detector by connecting a DC power supply in place of the generator, setting the meter to read FSD for a direct current equal to the peak RF current that flows when a generator is delivering 10 W into 50 Ω . Tests were then conducted on a variety of current transformers, with several different secondary load resistors; each time setting the generator output to give FSD of the meter monitoring the input current, and then setting the output level meter to read FSD by adjusting its series resistor. After setting, the output detector was then transferred to a DC power supply, and the voltage giving FSD, being equivalent to the peak value of the RF output voltage, was determined.

5 See, for example, **AC Electrical Theoy**, D W Knight. http://g3ynh.info/zdocs/AC_theory/ . Formula **18.3**.

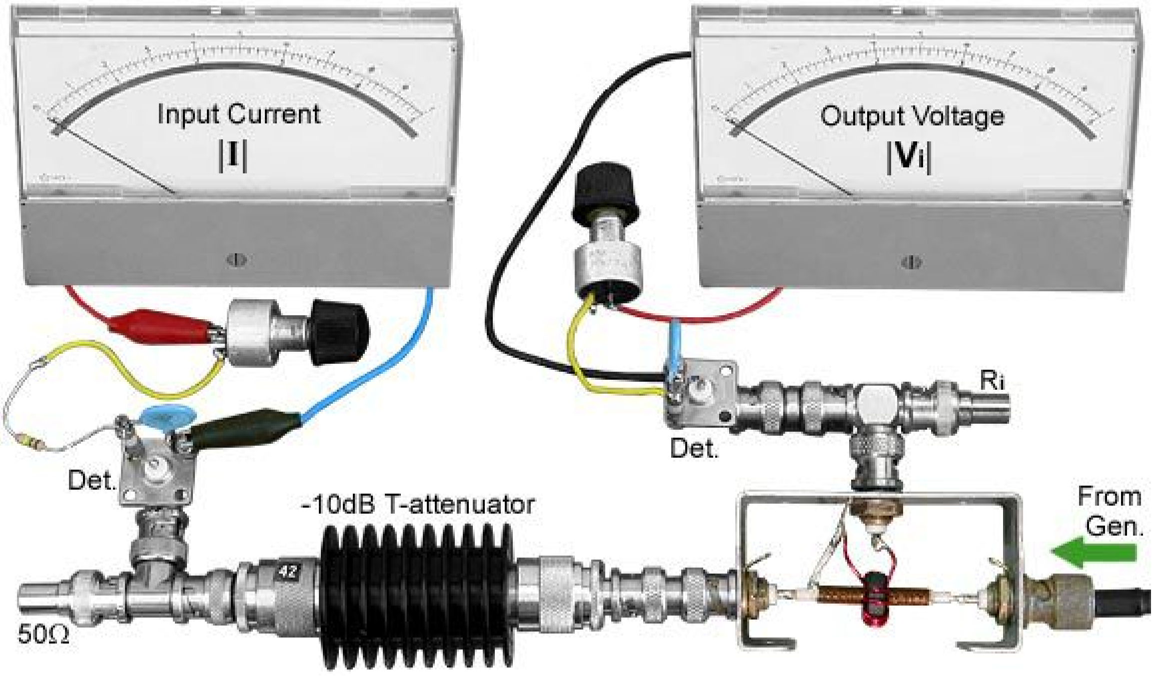


Fig 1. Early version of the test setup. In the final experiments, 10-turn potentiometers were used for the detector sensitivity adjustments and two AVO model 8 meters on their 50 μA ranges were used as indicators. The AVO 8 has a large accurately-linearised scale with an anti-parallax mirror.

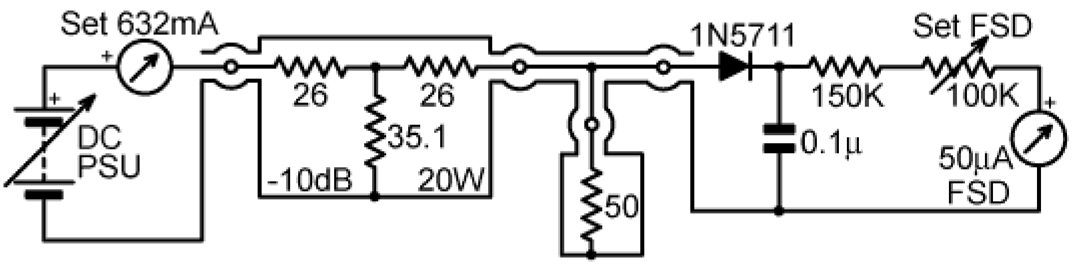


Fig 2. Establishing the reference current.

The initial current calibration step is illustrated in **fig. 2** above. The point in delivering direct current into the entire load assembly in this way is that it automatically takes into account the diode forward voltage drop and the tolerances of the resistors in the T-attenuator and the terminator. All we are interested in is the input current, and we do not care if the load presented to the transmitter is not exacly 50 Ω . The RMS current that flows when a generator is delivering exactly 10 W into exactly 50 Ω is:

$|\mathbf{I}| = \sqrt{(P/R)} = 447.2$ mA .

The equivalent peak current is:

$447.2\sqrt{2} = 632.4$ mA

and so this is the direct current that should be injected as accurately as is possible when setting the detector to read full-scale. Note however that typical bench power-supplies have a 0 - 30 V output range, whereas it requires 31.6 V to give 632 mA into a 50 Ω resistance. The solution to this problem is to put two power supplies in series (the author used a 13.8 V PSU in series with a 0 - 30 V PSU), having first checked that the terminals of the most positive power supply in the stack are floating with respect to ground. A further trick is to cook the whole assembly at an input current of 447 mA for 10 minutes or so, to raise its temperature to that which will be encountered during the RF measurements, and then briefly ramp the current to 632 mA for the setting of the meter series resistor. In this way the effect of thermal variations in resistance and diode forward voltage drop are minimised.

For the purpose of establishing the DC input current, the author had available two 3½ digit multimeters of different make but in a known good state of calibration. Consequently, although only one ammeter is shown in series with the PSU, both meters were placed in series so that the average could be taken. Fortuitously, the meters read 632 mA simultaneously, which gave confidence in both an emotional and a statistical sense. Both meters had a stated accuracy of ±0.5% ±1 mA on the ranges used, i.e., ±4.16 mA (the numbers after the decimal place are not significant), but by averaging the readings the uncertainty is reduced by a factor of $1/\sqrt{2}$. Hence the estimated standard deviation of the peak current setting was:

$4.16 / \sqrt{2} = 2.9$ mA

The peak current must of course then be divided by $\sqrt{2}$ to find the RMS current during the RF measurements, and the estimated standard deviation is scaled down accordingly. This gives an equivalent RMS input current of 446.9 ±2.1 mA. Finally, with much tapping of the case to jog the bearings, and careful use of the anti-parallax mirror, it was estimated that the detector current meter could be set to within ±0.2% of FSD. This corresponds to ±0.9 parts in 447 , and so it was established that RF measurements were made with a reference input current of 446.9 ±3 mA RMS.

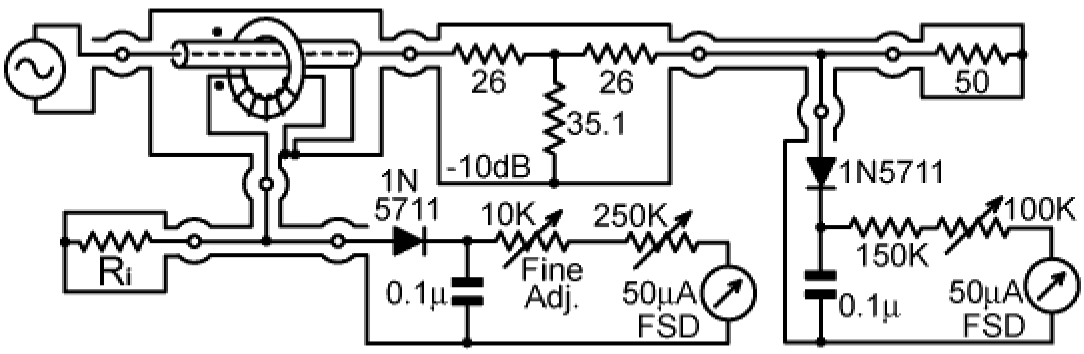


Fig 3. Setting the output detector meter series resistor.

The RF part of the procedure was conducted with the transformer under test installed as in the diagram above. In each case, the carrier level of the generator (radio transmitter) was turned up to give FSD of the input current meter, the output meter was set roughly to FSD by adjusting its series resistor, and the whole assembly was allowed to cook for about 10 minutes. With the system in thermal equilibrium, and much tapping of both meters to jog the bearings, the carrier level was then set exactly, and the series resistance of the output level meter was given its final adjustment. Precise adjustment of the transmitter carrier control was made possible by the use of an auxiliary reduction drive as shown in **fig 4.** below.

Fig 4.
Temporary reduction drive fitted to a transmitter carrier level control. Accurate setting of the power level is all but impossible without such an attachment. A 6:1 friction drive is shown coupled to the (outer) carrier control knob of a Kenwood TS930S HF transceiver by means of a short length of PVC tubing (warmed to soften it and then pushed on). An improvised stay for the reduction-drive body is attached to a lower cover retaining screw.

The immediate priority after switching off the transmitter was to measure the resistance of the transformer secondary load resistor before it had a chance to cool down. This was accomplished by having a DMM set to the correct range and ready with a good clean silver-plated BNC to 4 mm adapter installed. It was found possible to obtain a resistance reading within 3 seconds of switching off the RF input, and none of the resistances were seen to change on that timescale. This step was necessitated by the discovery that preliminary experiments had been invalidated by thermal variation of about 2% in some of the load resistors used. The meter used for the measurement had a stated accuracy of $\pm0.8\% \pm0.1\ \Omega$ on the range used. It gave the following readings when used to measure various 0.1% precision resistors mounted on good quality BNC plugs:

Reference resistor:	Short circuit	21.30 ±0.021	42.00 ±0.04	100.0 ±0.1
Meter reading:	0.1	21.4	42.1	100.1*5*

(the meter flickered between 100.1 and 100.2 when reading the 100 Ω resistor, so the last digit is given in italics to indicate that it was deduced). Evidently the accuracy of the meter was better than specified, but there is a need to subtract 0.1 Ω from all of its readings. The corrected resistance readings were assumed to have a standard deviation of no worse than $\pm0.2\ \Omega$.

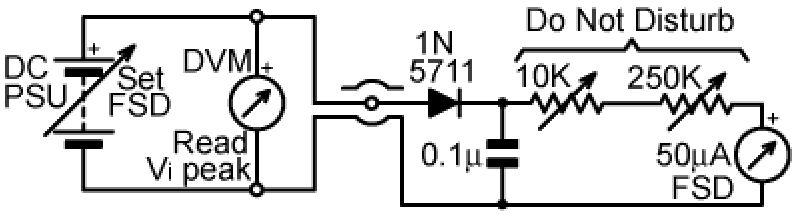


Fig 5. Reading the detector sensitivity to determine $V_{i(peak)}$

The final step in making a measurement was to determine the DC input voltage corresponding to FSD of the current transformer output detector, as shown in **fig. 5** above. The voltage obtained corresponds to the peak value of the current transformer output under RF conditions and will be given the symbol $V_{i(peak)}$. The DVM used for the measurement has an input resistance of 10 MΩ and gave the voltage of a Weston Standard Cell at 20°C to be 1.019 V on its 2 V range (exactly correct for an instrument of its resolution), and flickered between 1.01 V and 1.02 V on the 20 V range used. Its accuracy was therefore also better than specified, and the voltage measurements obtained were assumed to have a standard deviation no worse than ±0.02 V. To this however, must be added the resetting uncertainty of the detector meter, which was about ±0.2% for the RF setting step and ±0.2% for the DC measurement step, giving 0.3% overall. Hence detector voltage measurements were made with an uncertainty of ±0.02 V ±0.3% , the two sources of error being uncorrelated [6]. This uncertainty is scaled down by a factor of $1/\sqrt{2}$ when we convert from peak to RMS values.

Some readers might question the need to read the output voltage by applying DC to the detector, since it is 'obvious' that the voltage can be read by connecting a DVM across the smoothing capacitor while the transmitter is running. It was found however, that DVM readings in the presence of an operating radio transmitter could not be trusted. The author saw readings that were in error by a factor of as much as 2.2 (reading 22 V instead of 10 V) when this was tried. This problem might be solved by feeding the DVM via an efficient low-pass filter, but apparent plausibility of the readings is the only measure of success in such a case. The DC method also takes the diode forward voltage drop into account automatically, whereas a direct reading must be corrected. Hence, any improvement in accuracy engendered by a direct reading will be partially negated by uncertainties in the diode model used. Any error in the diode correction will moreover be systematic (i.e., it will introduce bias into all of the results obtained), whereas the setting and resetting errors of a properly zeroed moving-coil meter are largely random.

As mentioned previously, the need to control random errors very carefully arises because transformer efficiency is high. We must also be aware however, that there might be residual systematic errors; one being due to the possibility that there might be detector inefficiency beyond that associated with the diode forward voltage drop under static (DC) conditions. The issue here is that when a diode conducts under dynamic conditions, the current occurs in pulses that are considerably larger than the average current. Thus there might be a greater effective diode forward drop than has been allowed for by making DC settings and measurements. The author's partial solution to this problem was to load the detectors lightly by using 50 µA FSD meters, and to choose some of the test transformer and load resistor combinations to give output voltages comparable to the voltage applied to the input current detector. The point in the latter case is that when the RMS voltages applied to the two detectors are about the same, the effects of detector inefficiency are cancelled. In the event that the voltages at the two detectors are widely different however, we must be aware of detector inefficiency as a possible additional source of error.

4. Detector loading

The output voltage detector requires a small amount of power to drive it. Consequently, the effective current transformer load resistance is very slightly lower than that obtained by measuring the resistor. The power consumed by the detector is the same as that which is required to deflect the meter to full-scale when a DC supply is connected to it, i.e., it is given by:

$$P_{\mathbf{det}} = V_{\mathbf{i(peak)}} \times I_{\mathbf{fsd}}$$

This power can be converted into an equivalent parallel load resistance $R_{\mathbf{det}}$ using:

$$P_{\mathbf{det}} = |\mathbf{V}_{\mathbf{i(meas)}}|^2 / R_{\mathbf{det}}$$

6 See, for example, **Scientific Data Analysis**, D W Knight, http://g3ynh.info/zdocs/math/data_analy.pdf .

where $|\mathbf{V}_{i(meas)}|$ is the 'measured' RMS transformer output and is given by:

$$|\mathbf{V}_{i(meas)}| = V_{i(peak)} / \sqrt{2}$$

Hence:

$$R_{det} = |\mathbf{V}_{i(meas)}|^2 / (V_{i(peak)} I_{fsd})$$

$$R_{det} = (V_{i(peak)} / \sqrt{2})^2 / (V_{i(peak)} I_{fsd})$$

$$R_{det} = V_{i(peak)} / (2 I_{fsd})$$

The effective current-transformer load resistance is thus:

$$R_i' = R_i // R_{det}$$

Hence:

$$R_i' = 1 / [(1/R_i) + (2 I_{fsd} / V_{i(peak)})]$$

For the nominal 50 µA meter used, I_{fsd} was measured to be 52.0 ±0.36 µA. Hence:

$R_i' = 1 / [(1/R_i) + (0.000104 / V_{i(peak)})]$	[Ohms]	**1**

R_{det} is always much larger than R_i. Hence the contribution to the uncertainty in R_i' from the uncertainty in R_{det} will be negligible in comparison to the contribution from the uncertainty in R_i. Hence the estimated standard deviation of R_i' can be taken to be the same as that of R_i (i.e., ±0.2 Ω). In practice, the use of R_i' in place of R_i is a minor correction, increasing the calculated efficiencies by between +0.0004 and +0.0020 in this study, but the adjustment is nonetheless worthwhile.

5. Data analysis

As was discussed above, the output voltage of a current transformer, according to the 'ideal transformer with secondary reactance' model, is given by the expression:

$|\mathbf{V}_{i\,(theor)}| = |\mathbf{I}|\, R_i' / \{ N \sqrt{[1 + (R_i' /X_i)^2]} \}$	[Volts]	**2**

This form was chosen because it has a separable reactance correction factor $1/\sqrt{[1 + (R_i' / X_i)^2]}$, which is very close to unity at 30 MHz. We will include this factor when calculating theoretical values, but since its contribution is small, we can ignore it when determining the uncertainty in the calculated $|\mathbf{V}_i|$ due to the uncertainties in the values of $|\mathbf{I}|$ and R_i'. Hence, for the purposes of estimating the standard deviations of the calculated $|\mathbf{V}i|$ values:

$$|\mathbf{V}_{i\,(theor)}| = |\mathbf{I}|\, R_i' / N$$

The way in which the errors in two or more variables combine to determine the error in the output of a formula using those variables is explained elsewhere[7]. Two methods for estimating the standard deviation of the output quantity are available, one (numerical) involving parameter shifting, the other (analytical) involving calculus. Here we will use the analytical method because it will simplify things greatly if we have the error function as an algebraic expression. Since the errors in $|\mathbf{I}|$ and R_i' are uncorrelated, the error in $|\mathbf{V}_{i\,(theor)}|$ is given by the orthogonal vector sum of the rate of change of the formula with respect to $|\mathbf{I}|$ multiplied by the uncertainty in $|\mathbf{I}|$ and the rate of change of the formula with respect to R_i' multiplied by the uncertainty in R_i', i.e.:

$$\sigma_{\mathbf{vi\ theor}} = \sqrt{\{\ [\ \sigma_{|\mathbf{I}|}\ \partial|\mathbf{V}_{i\,(theor)}| / \partial|\mathbf{I}|\ ]^2 + [\ \sigma_{\mathbf{Ri}}\ \partial|\mathbf{V}_{i\,(theor)}| / \partial R_i'\ ]^2\ \}}$$

where σ represents the estimated standard deviation of the quantity indicated by its subscript, and ∂ indicates a partial differential (i.e., differentiation of one quantity with respect to another with all other variables held constant is implied). The differentiations are trivial in this case:

$$\partial|\mathbf{V}_{i\,(theor)}| / \partial|\mathbf{I}| = R_i' / N$$

$$\partial|\mathbf{V}_{i\,(theor)}| / \partial R_i' = |\mathbf{I}| / N$$

Hence:

$$\sigma_{\mathbf{vi\ theor}} = \sqrt{[\ (\ \sigma_{|\mathbf{I}|}\ R_i' / N\ )^2 + (\ \sigma_{\mathbf{Ri}}\ |\mathbf{I}| / N\ )^2\ ]}$$

which simplifies to:

$$\sigma_{\mathbf{vi\ theor}} = (1/N) \sqrt{[\ (\ \sigma_{|\mathbf{I}|}\ R_i'\ )^2 + (\ \sigma_{\mathbf{Ri}}\ |\mathbf{I}|\ )^2\ ]} \qquad \text{[Volts RMS]}$$

From the previous discussion we have: $|\mathbf{I}| = 0.4469$ A , $\sigma_{|\mathbf{I}|} = 0.003$ A , and $\sigma_{\mathbf{Ri}} = 0.2\ \Omega$. The other quantities vary between experiments. Hence our error function is:

$\sigma_{\mathbf{vi\ theor}} = (1/N) \sqrt{[\ (\ 0.003\ R_i'\ )^2 + (\ 0.2 \times 0.4469\ )^2\ ]}$ [Volts]	**3**

The object of the exercise is to measure the transformer efficiency, as defined by the expression:

$$k' = |\mathbf{V}_{i\,(measured)}| / |\mathbf{V}_{i\,(theor)}|$$

Previously, we determined the uncertainty of the measured RMS output voltage to be $\pm 0.02/\sqrt{2}$ V $\pm 0.3/\sqrt{2}$ % from two uncorrelated error sources. To use these numbers, we must first convert the percentage into Volts, and then combine them as orthogonal vectors. Hence (abbreviating "measured" to "meas"):

$\sigma_{\mathbf{vi\ meas}} = (1/\sqrt{2})\sqrt{[\ 0.02^2 + (0.003\ |\mathbf{V}_{i\,(meas)}|\ )^2\ ]}$ [Volts]	**4**

Now, having estimated standard deviations for both the measured and the theoretical values of $|\mathbf{Vi}|$, and assuming them to be uncorrelated, we have:

$$\sigma_{\mathbf{k'}} = \sqrt{\{\ [\ (\sigma_{\mathbf{vi\ meas}}\ \partial k' / \partial|\mathbf{Vi}_{(meas)}|\ ]^2 + [\ (\sigma_{\mathbf{vi\ theor}}\ \partial k' / \partial|\mathbf{V}_{i\,(theor)}|\ ]^2\ \}}$$

7 See, for example **AC Electrical Theory** (DWK, already cited), **section 39**, or **Scientific Data Analysis** (DWK, already cited).

where:

$\partial k' / \partial|\mathbf{V}_{i\,(meas)}| = 1/ |\mathbf{V}_{i\,(theor)}|$

and:

$\partial k' / \partial|\mathbf{V}_{i\,(theor)}| = -|\mathbf{V}_{i\,(meas)}| / |\mathbf{V}_{i\,(theor)}|^2$

hence:

$\sigma_{k'} = \sqrt{\{ [\sigma_{vi\,meas} / |\mathbf{V}_{i\,(theor)}|]^2 + [\sigma_{vi\,theor} |\mathbf{V}_{i\,(meas)}| / |\mathbf{V}_{i\,(theor)}|^2]^2 \}}$	[dimensionless]	5

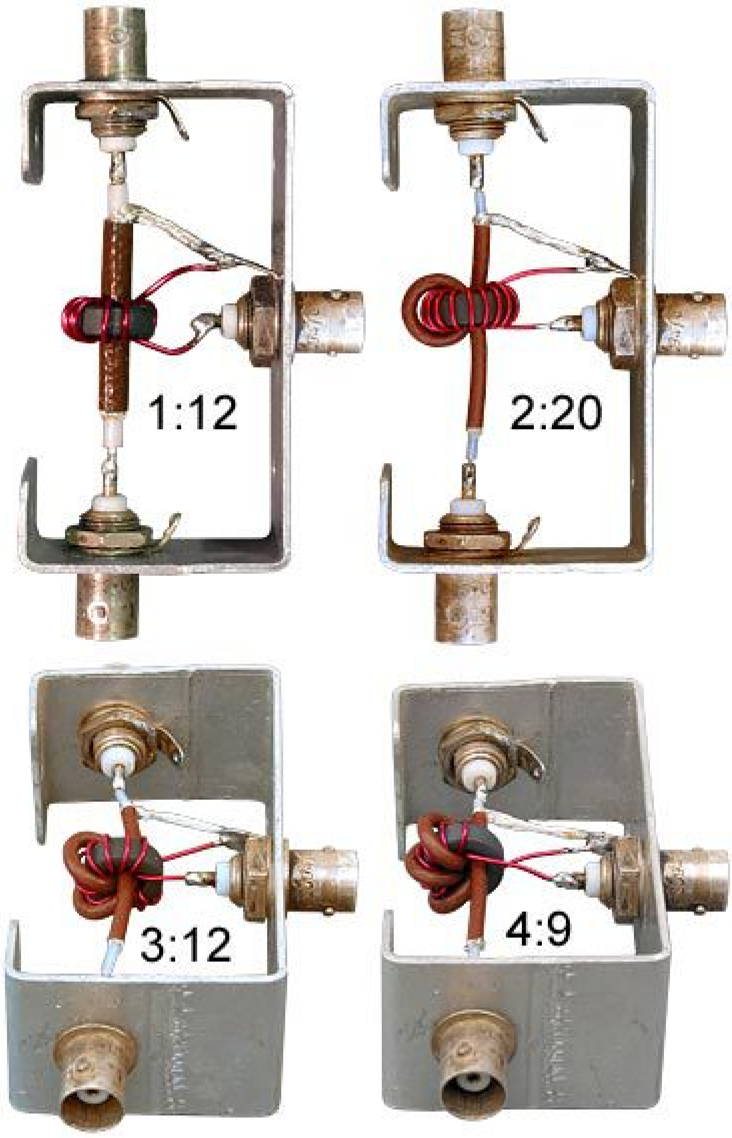


Fig **6.**

Current transformers with 1, 2, 3 and 4 turn primary windings used in the transfer efficiency tests described in the text. For a toroidal transformer, one turn is equivalent to one pass through the hole. All transformers are wound on Amidon (Fair-Rite) FT-50 ½" (12.7 mm) diameter beads.

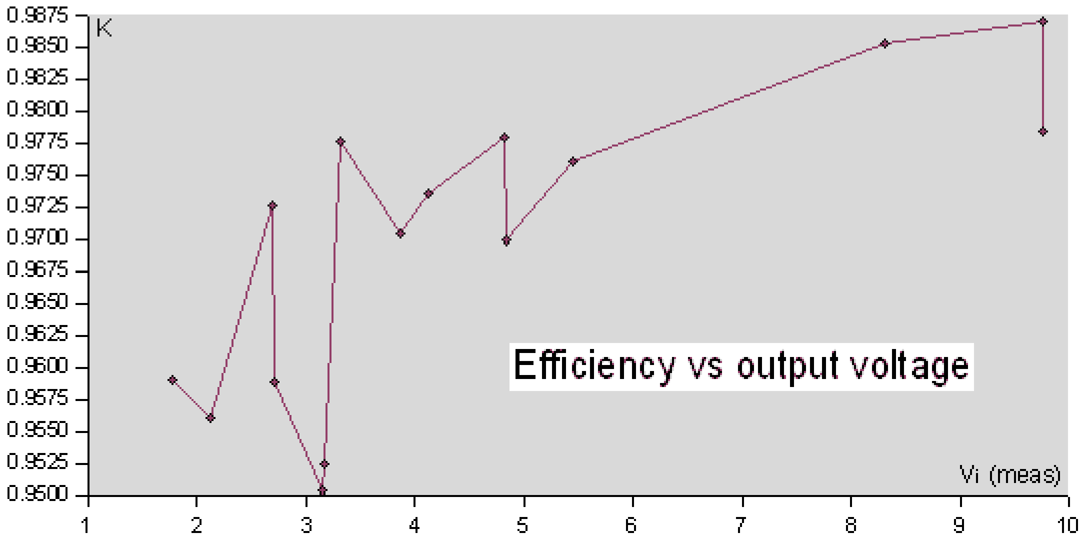
6. Measurements

Detais of the calculations can be found in the open-document spreadsheet **Itr_k.ods**. Shown in the screen-capture image below are the results of measurements made on five different current transformers with load resistances ranging between 90 Ω and 25 Ω . The generator frequency was 30 MHz in all cases. Column A is the test transformer identified by its core material and turns (N_{pri} : N_{sec}) . Column B is the turns ratio, defined as $N = N_{sec} / N_{pri}$. Column C is the secondary inductance in μH measured at 1.5915 MHz. Column D is the secondary parallel capacitance, assumed to be about 2 pF for the input capacitance of the 1N5711 detector diode, plus 2 pF for the short length of unmatched transmission line leading to the detector. Altering this capacitance by ±3 pF affects only the 4th decimal place of the calculated efficiency, and so its exact value is not important. Column E is the load resistance measured within 3 seconds of switching off the RF power. Column F is the effective load resistance given by equation (**1**). Column G is the theoretical output voltage given by equation **(2)**, and column H is its estimated standard deviation (ESD) given by equation **(3)**. Column I is the measured peak value of the output voltage obtained by applying DC to the detector. Column J is the 'measured' RMS value obtained by dividing the peak value by √2 , and column K is its ESD as given by equation **(4)**. Column L is the transfer efficiency k', and column M is its ESD calculated using equation **(5).** Note that there are two measurements on the 1:12 transformer with an 89.5 Ω load. One of these was performed early in the experimental run, and one was performed towards the end as a test of reproducibility.

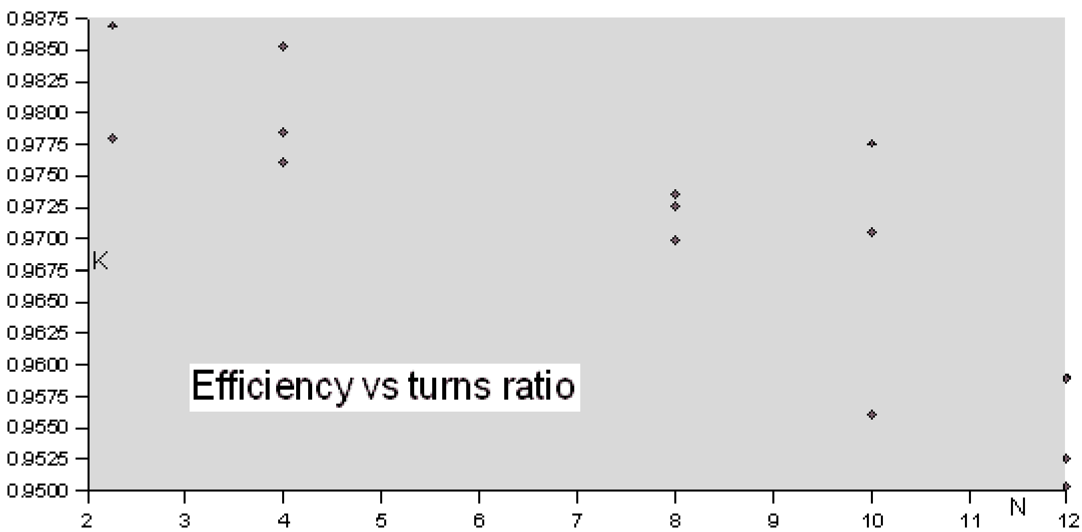
	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Xformer	N	Li	Cis	Ri	Ri'	Vi rms	esd	Vi pk	Vi rms	esd	Effy	esd
2	FT50-		/ uH	/ pF			theor	±	meas	meas	±	k'	±
3	61, 1:12	12.00	8.45	4.0	89.5	89.31	3.3259	0.0235	4.47	3.1608	0.0157	0.9503	0.0082
4	61, 1:12	12.00	8.45	4.0	89.5	89.31	3.3259	0.0235	4.48	3.1678	0.0157	0.9525	0.0082
5	61, 1:12	12.00	8.45	4.0	76.2	76.04	2.8318	0.0204	3.84	2.7153	0.0153	0.9589	0.0088
6	61, 1:12	12.00	8.45	4.0	49.8	49.70	1.8507	0.0145	2.51	1.7748	0.0146	0.9590	0.0109
7	67, 2:20	10.00	7.50	4.0	89.5	89.35	3.9928	0.0283	5.48	3.8749	0.0164	0.9705	0.0080
8	67, 2:20	10.00	7.50	4.0	76.2	76.07	3.3995	0.0245	4.70	3.3234	0.0158	0.9776	0.0084
9	67, 2:20	10.00	7.50	4.0	49.9	49.81	2.2261	0.0174	3.01	2.1284	0.0148	0.9561	0.0100
10	61, 1:8	8.00	4.07	4.0	89.5	89.38	4.9868	0.0353	6.84	4.8366	0.0175	0.9699	0.0077
11	61, 1:8	8.00	4.07	4.0	76.1	76.00	4.2416	0.0306	5.84	4.1295	0.0166	0.9736	0.0080
12	61, 1:8	8.00	4.07	4.0	49.8	49.73	2.7771	0.0217	3.82	2.7011	0.0153	0.9727	0.0094
13	61, 3:12	4.00	8.95	4.0	89.4	89.34	9.9802	0.0706	13.81	9.7651	0.0251	0.9784	0.0074
14	61, 3:12	4.00	8.95	4.0	75.6	75.55	8.4400	0.0609	11.76	8.3156	0.0226	0.9853	0.0076
15	61, 3:12	4.00	8.95	4.0	49.9	49.87	5.5710	0.0436	7.69	5.4377	0.0182	0.9761	0.0083
16	61, 4:9	2.25	5.20	4.0	49.8	49.78	9.8866	0.0774	13.80	9.7581	0.0251	0.9870	0.0081
17	61, 4:9	2.25	5.20	4.0	24.8	24.79	4.9237	0.0517	6.81	4.8154	0.0174	0.9780	0.0109

7. Interpretation

As mentioned earlier, one possible source of systematic error is detector inefficiency. If such an experimental defect were present, it would manifest itself as a tendency to produce pessimistic estimates of transformer efficiency whenever the voltage at the output detector is low in comparison to the voltage at the input current detector. For the purpose of examining this possibility, a plot of transformer efficiency against measured secondary voltage is shown below:



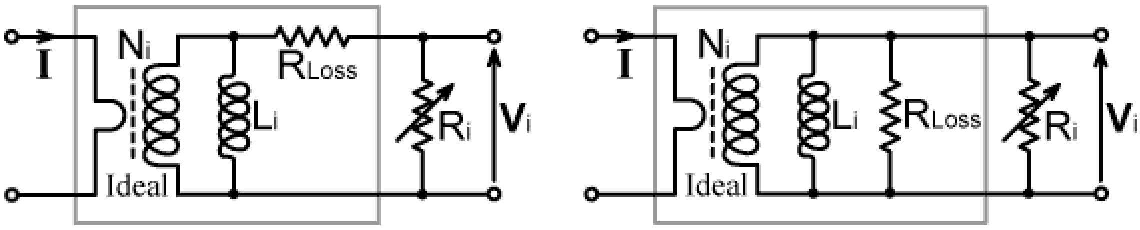
It might appear at first glance that there is an upward trend in the efficiency as the output voltage increases. The behaviour is however, also chaotic, and were we to fit the graph to a regression line and use the resulting function to correct the data, it would have the unfortunate consequence of making some of the transformers appear to be more than 100% efficient. More reasonably we should note that it is the transformers with low turns ratio which give the greatest output, which means that a tendency for the efficiency to fall as the turns ratio increases would produce a similar correlation. The scatter diagram shown below examines this alternative.



Here there is an ordered trend. There are also sound physical reasons for expecting the efficiency to improve as the turns ratio is reduced, which is that there will be a relative reduction in primary leakage inductance. Hence we should reject the detector inefficiency hypothesis and conclude that low-ratio current transformers are more efficient than high-ratio ones.

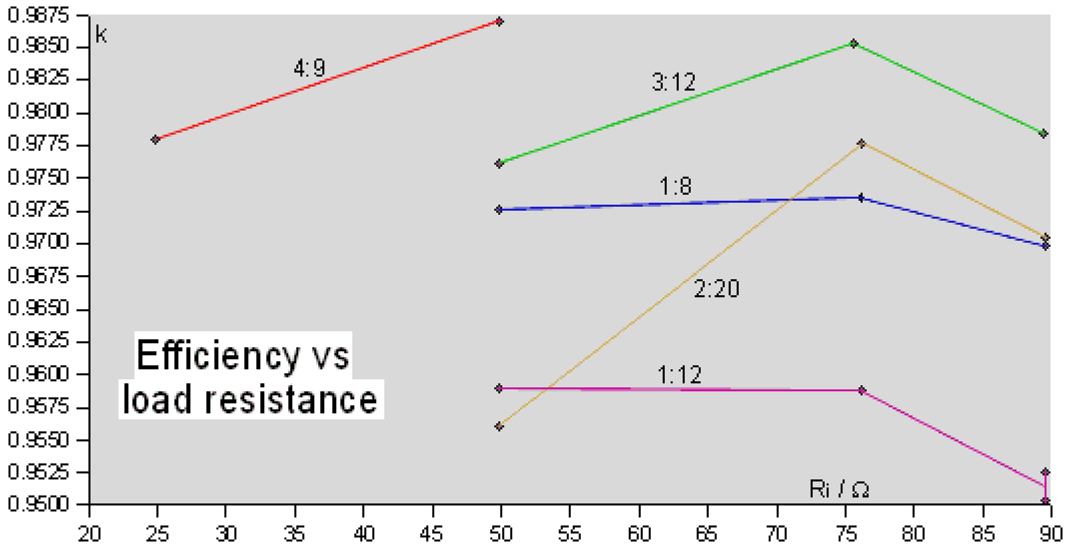
Now consider the two transformer models shown below in relation to the plot of transformer efficiency versus load resistance shown below them:

Fig 9. Candidate current transformer models with less than 100% transfer efficiency.



Series loss component.
Efficiency increases as R_i increases.

Parallel loss component.
Efficiency decreases as R_i increases.



Unfortunately, there are insufficient data to permit a clear statistical distinction to be made between the two models, but if the plot is considered as a scatter diagram there is an apparent downward trend as the load resistance increases. There is also a sound physical reason for favouring the parallel loss model, which is that the loss resistance is identifiable in part with the core loss referred to the transformer secondary. Hence, bearing in mind that any simplification applied to a component model reduces its accuracy, we can conclude that it is reasonable to account for the shortfall in output of an 'ideal current transformer with secondary reactance' by invoking a parasitic parallel resistance R_k, which is defined as:

$$R_k = k' R_i / (1 - k')$$

The experimental results produced during this investigation are summarised below. The broad conclusion is that small RF current transformers wound on ferrite beads have transfer efficiencies in the region of 95 to 99%. Transformers of low turns ratio are more efficient than transformers of high turns ratio. Provided that the winding resistance is low, the transfer loss can be considered to be due to a parallel parasitic resistance.

Measured efficiency (k') factors for a selection of RF current transformers.
Standard deviations are expressed in brackets after the number as uncertainty in the last digit.

Core Type	Turns	L_i / µH	**Load resistance** R_i / Ω			
			25	50	75	90
FT50-61	1:12	8.45 (21)	-	0.959 (11)	0.959 (9)	0.951 (8)
FT50-61	1:8	4.07 (10)	-	0.973 (10)	0.974 (8)	0.970 (8)
FT50-67	2:20	7.50 (19)	-	0.956 (10)	0.978 (9)	0.971 (8)
FT50-61	3:12	8.95 (22)	-	0.976 (9)	0.985 (8)	0.978 (7)
FT50-61	4:9	5.20 (13)	0.978 (12)	0.987 (8)	-	-

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