On the effects of dielectric or permeable formers on the inductance and self-capacitance of solenoids

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Attached below are the formulas for calculating the effects of dielectric (ε >1) and or permeable (μ >1) formers on the inductance and self-capacitance of solenoids.

Assumptions: All length scales << wavelength λ. In the exceptional case of self-resonance in which the wire length $l_w \approx \lambda$ then the transmission line model is used, but all other dimensions remain << λ

1) For a cylindrical hollow former of thickness d_a and $\varepsilon = \varepsilon_1$, the change in dielectric constant or permeability is given by the Maxwell-Garnett formula which is in fact exact for cylinders (neglecting edge effects) and concentric spheres. This was proved by a clever energy argument due to Hashin and Shtrikman in 1962 [1]. Hence the effective dielectric constant $\bar{\varepsilon}$ is given by:

$$\bar{\varepsilon} = 1 + \frac{\eta}{\frac{1}{(\varepsilon_1 - 1)} + \frac{1}{P}(1-\eta)}. \qquad (1)$$

Here η is the volume fraction = 4 d_a /D for infinitely long cylinders where D is the diameter of the former. Here P=2 for cylinders while P=3 is for spheres see later. In other words a hollow former is equivalent to a solid former whose dielectric constant is given by eqn(1) . Note that it has the correct limit when $\eta = 0 \; or \; 1$. In the case of magnetic permeable formers simply replace ε by μ. Incidentally a straightforward volume fraction averaging such as used earlier by David Knight will only give an upper bound which tends to exaggerate the effective dielectric constant $\bar{\varepsilon}$ see eqn(1). It does not even work for low fractions and is related to a well known theorem proved by Wiener in 1912 [2], which states that $\bar{\varepsilon}$ is bounded from below by <1/ ε> and from above by <ε>, where <> denote volume averages, see Chapter 2 of ref [3].

This formula is sufficient for low frequency (LF) applications and can be substituted in the standard LF inductance or proximity capacitance formulas. For HF close to self-resonance (SRF) the transmission line model is used. In this case the effective dielectric constant close to the surface of the former needs to be considered since this is the region where most of the field is concentrated. For this purpose we can use the Lorentz local field formula to correct for the effect of the dielectric constant of the former on the local field. Again this formula is exact for cylinders neglecting edge effects or spheres but approximate otherwise. It can also be obtained from an exercise in Smythe's book on Classical Electricity [4]. So for the transmission line model use $\bar{\bar{\varepsilon}}$ as the effective dielectric constant this is given by:

$$\bar{\bar{\varepsilon}} = 1 + \frac{\bar{\varepsilon} - 1}{\bar{\varepsilon} + (P-1)} \left(\frac{D + d_w}{D}\right)^P \qquad (2)$$

where $\bar{\varepsilon}$ is given in eqn(1) and d_w is the wire thickness.

2) **Edge effects**.

This could possibly be tractable analytically for cylinders of finite length by conformal mapping methods but it will be extremely messy. For narrow long coils one can use the above formulas and substitute for η the relevant volume fraction and for P the appropriate polarisation factors for ellipsoids or spheroids with high eccentricity. These factors have been worked out by Osborn [5] and Stoner [6] and involve elliptic integrals in 1940s strangely much later than Nagaoka for coils. For broad and short coils, I think one can do quite well using the formula for spheres P=3.

3) **Some Thoughts**: The above formulas assume that the coil circumference is at a constant potential, valid in the quasi-static limit $D << \lambda$. At microwave frequencies when this breaks down, assuming that one has calculated a prior the potential distribution $V(\theta)$, the model can be improved by taking into account multipole corrections. This is similar to a line charge away from the cylinder or point charge away from the sphere whose potential $V(\theta)$ on the surface is no longer uniform, which leads to a multipole expansion beyond the dipole ones given above. An alternative is to use Kelvin's method of images, which from past work [7], was found to converge much better than the multipole series. Smythe's book [4] also has some such examples taken from past year Cambridge Tripos exam questions.

A note of caution is necessary: The use of these formulas should be viewed as no more than a good initial guide to further experimentation. It would be good if their empirical status can be further established or better fitted via a parameter such as P. Unfortunately experiments with a variety of formers are rather tedious, but the results highly useful as a handbook on coil formers for example. The above formulas are obviously not suitable for magnetic core formers due to hysteresis and other non-linear effects.

References:

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[6] Stoner, E.C. , Philosophical Magazine **36**, 803-6 (1945).

[7] Choy T.C., Alexopoulos A .and Thorpe M.F., Proceedings of the Royal Society of London **A454**, 1993-2013 (1998).