The Maximally-Flat Current Transformer[1]

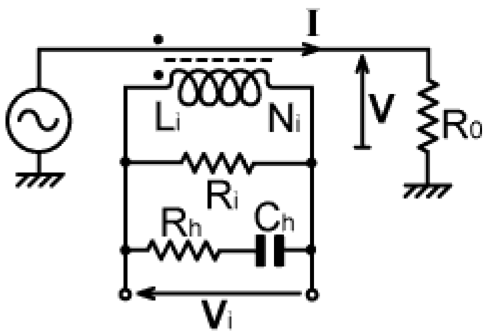
By David W Knight

In the design of current transformers for broadband RF applications, good performance at the low-frequency end of the operating range requires either, a large secondary inductance, or a low value of secondary load resistance, or both. Large inductance however implies either the use of high-permeability core material; with attendant core losses and dispersive effects (variation of A_L with frequency); or a large number of turns, with associated low sensitivity and high winding resistance. A low value of load resistance also implies low sensitivity; and we are in danger of concluding that we can make excellent broadband transformers as long as we don't want any actual output.

There is however, a way of obtaining a flat amplitude response with high sensitivity by steepening the skirt of the low-frequency band-edge. The downside of the technique is that it degrades the phase-performance, but it is nevertheless perfectly suitable for phase-insensitive instruments such as ammeters. It involves placing a capacitor between the transformer output and the load resistor, and will be referred to here as the ***maximally-flat current transformer***.

The maximally-flat current transformer is an offshoot of a mathematical analysis[2] carried out by this author (DWK) for the purpose of evaluating a low-frequency compensation scheme patented by Will Herzog in 1988 [**US Pat. No. 4739515**]. The maximally-flat configuration was however, not considered by Herzog and is not subject to his patent. Herzog's idea was that, since the output of the transformer falls at low frequencies, but can be made to increase by increasing the secondary load resistance; it ought to be possible to achieve a correction by splitting the load resistance into two parallel components and placing a capacitor in series with one of those. The point is that the output of the transformer will increase and undergo a negative phase-shift as the reactance of the capacitor increases, and it might be possible to tailor this effect to compensate for the falling output and positive phase shift caused by the falling inductive reactance. Neither the patent nor an associated article[3] gave any analysis or design equations however, which is why the investigation referred-to above was carried out.

The outcome of the analysis was that Herzog's compensation circuit (shown right) can be used to improve the phase performance of the transformer at the expense of degrading the amplitude performance. It therefore has little merit in comparison to the standard practice in bridge design, which is to tailor the amplitude and phase response of the voltage sampling network to match that of the current transformer. The spurious amplitude error however, is in the form of a large low-frequency hump, and it was discovered on further analysis that this could be eliminated by taking the output from the load-resistor side of the capacitor.

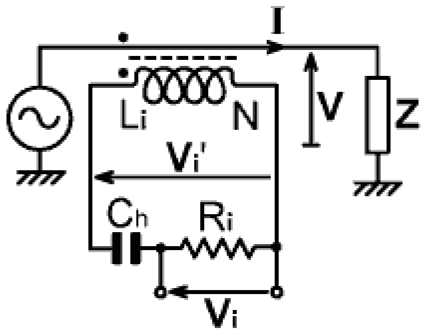


Herzog's LF phase compensation

1 Version 1.00, 20th Feb. 2014. © D. W. Knight, 2014 (Updated version of an HTML article first published in 2008). Please check the author's website to make sure you have the most recent version of this document and the accompanying spreadsheet file: http://www.g3ynh.info/ .

2 **Analysis of Herzog's LF phase compensation method.** D W Knight. http://g3ynh.info/zdocs/bridges/ .

3 **VSWR Bridges**, Will Herzog K2LB, Ham Radio, March 1986, p37-40.



Maximally-flat network

For the modified circuit shown on the right, we can write an expression for the voltage appearing across the secondary winding as follows:

$$\mathbf{V_i}' = (\,\mathbf{I}/N\,)\,[\;\mathbf{j}X_{Li} // (R_i + \mathbf{j}X_{Ch})\,]$$

$$= (\,\mathbf{V}/\mathbf{Z}N\,)[\;\mathbf{j}X_{Li} // (R_i + \mathbf{j}X_{Ch})\,]$$

The output voltage $\mathbf{V_i}$ however is obtained from $\mathbf{V_i}'$ via a potential divider composed of $\mathbf{j}X_{Ch}$ and R_i, hence:

$$\mathbf{V_i} = \mathbf{V_i}'\, R_i / (R_i + \mathbf{j}X_{Ch})$$

$$= (\mathbf{V}/\mathbf{Z}N)\,[\,\mathbf{j}X_{Li} //(R_i + \mathbf{j}X_{Ch})\,]\, R_i /(R_i + \mathbf{j}X_{Ch})$$

which, by expanding the parallel product, gives:

$$\mathbf{V_i} = (\mathbf{V}/\mathbf{Z}N)\,[\,\mathbf{j}X_{Li}(R_i + \mathbf{j}X_{Ch}) / (\mathbf{j}X_{Li} + R_i + \mathbf{j}X_{Ch})\,]\, R_i / (R_i + \mathbf{j}X_{Ch})$$

and has the corresponding dimensionless transfer function:

$$\mathbf{V_i} / \mathbf{V} = \boldsymbol{\eta}_i = [\,\mathbf{j}X_{Li} / (\mathbf{j}X_{Li} + R_i + \mathbf{j}X_{Ch})\,]\, R_i / N\,\mathbf{Z}$$

This, as is usually the case, is easier to handle in reciprocal form:

$$1/\boldsymbol{\eta}_i = [\,(\mathbf{j}X_{Li} + R_i + \mathbf{j}X_{Ch}) / \mathbf{j}X_{Li}\,]\, N\,\mathbf{Z} / R_i$$

Which rearranges to:

$$1/\boldsymbol{\eta}_i = [1 + (X_{Ch} / X_{Li}) - \mathbf{j}R_i / X_{Li}\,]\, N\,\mathbf{Z} / R_i$$

Now we can imagine that the network is part of a bridge that, at infinite frequency, balances when $\mathbf{Z} = R_0$. In this limit, $X_{Li} \rightarrow \infty$ and $X_{Ch} \rightarrow 0$, and so the reciprocal transfer function reduces to:

$$1/\eta_\infty = N\,R_0 / R_i$$

This can be equated to the actual transfer function when $\mathbf{Z}$ is chosen to balance our hypothetical bridge, i.e.,

$$N\,R_0 / R_i = [\,1 + (X_{Ch} / X_{Li}) - \mathbf{j}R_i / X_{Li}\,]\, N\,\mathbf{Z} / R_i$$

Hence:

$R_0 / \mathbf{Z} = [\,1 + (X_{Ch} / X_{Li}) - \mathbf{j}R_i / X_{Li}\,]$	1

This expression captures all of the magnitude and phase errors of the current transformer network and so characterises the circuit. In this case however, we are interested in finding the component relationships that give the maximally flat amplitude response, and to this end we take the magnitude:

$\lvert R_0 / \mathbf{Z} \rvert = \sqrt{\{ (1 + X_{Ch} / X_{Li})^2 + (R_i / X_{Li})^2 \}}$	**2**

Now, factoring $1 / X_{Li}^2$ from the square-root bracket and expanding we have:

$$\lvert R_0 / \mathbf{Z} \rvert = (1/ X_{Li}) \sqrt{\{ X_{Li}^2 + X_{Ch}^2 + 2X_{Li} X_{Ch} + Ri^2 \}}$$

i.e.,

$$\lvert R_0 / \mathbf{Z} \rvert = (1/ X_{Li}) \sqrt{\{ X_{Li}^2 + X_{Ch}^2 + [R_i^2 - 2 L_i / C_h] \}}$$

What to do next to find the maximally flat amplitude condition is not immediately obvious, but on the basis that such a condition should exist, the circuit was simulated to see what would happen as C_h was varied. The solution was quickly discovered once the model had been set up, and centres on the pair of terms captured between square brackets in the expression above. If we multiply $1/ X_{Li}$ back into the square-root bracket we have:

$$\lvert R_0 / \mathbf{Z} \rvert = \sqrt{\{ 1 + (X_{Ch} / X_{Li})^2 + [R_i^2 - 2 L_i / C_h] / XL_i^2 \}}$$

It transpires that, for L_i of around 10 μH and C_h of a few nF, the term $(X_{Ch} / X_{Li})^2$ is much smaller than 1 for frequencies of 1 MHz and above. Hence, the variation of amplitude with frequency can be effectively eliminated over a very wide frequency range by choosing components so that:

$R_i^2 - 2 L_i / C_h = 0$	**3**

In the analysis of Herzog's LF compensation method, it was found that best phase performance could be obtained by choosing the network components to give the condition of critical damping; i.e., with the network on the borderline between ability to exhibit real resonance or imaginary resonance [4]. It transpires here however, that the capacitance for maximum flatness is exactly twice the capacitance for critical damping. This places the network state well within the imaginary resonance region, as might be expected from the requirement that there will be no peak in the response. It also means that the phase response will never coincide with the zero axis at finite frequency (neglecting the transformer propagation delay); but neither does the phase response of the uncompensated network, and we can deal with that issue in situations where phase is important by modifying the companion voltage sampling network to have the same response.

The phase angle of **Z** can be determined from equation **(1)** as the arctangent of the *negative* of imaginary part divided by the real part (the phase angle of 1/**Z** is the equal and opposite of the phase angle of **Z**), i.e.,

$$\tan\varphi = (R_i /X_{Li}) / [1 + (X_{Ch} / X_{Li})]$$

which rearranges to:

$\tan\varphi = R_i / (X_{Ch} + X_{Li})$	**4**

Notice that when $X_{Ch} + X_{Li} \rightarrow 0$, $\tan\varphi \rightarrow \infty$ and $\varphi \rightarrow +90°$. An infinitesimal reduction in frequency will then cause the tangent to change sign. This, according to the default behaviour of

4 **AC Electrical Theory**, D W Knight. http://g3ynh.info/zdocs/AC_theory/ . See **section 21**.

spreadsheets, calculators and other mathematical engines, may make it appear that the phase angle has suddenly switched to -90°, but this phase discontinuity does not actually occur. An ambiguity arises because inverse trigonometric functions have an infinite number of solutions for every argument. In the case of tangents:

$\mathrm{Tan}\varphi = \mathrm{Tan}(\varphi \pm n \times 180°)$

where n is any whole number including zero. E.g., Tan(91°) is the same as Tan(-89°). Hence, there is no unique solution to the inverse tangent function; and so, strictly:

If $x = \mathrm{Tan}\varphi$

Then $\varphi = \mathrm{Arctan}(x) \pm n \times 180°$

Hence, in this case, we should interpret the change in sign of the tangent, not as a switch to a negative angle, but as the angle moving out of the first (0 to 90°) quadrant into the second (90 to 180°) quadrant. The frequency at which this event occurs is given by the standard resonance formula:

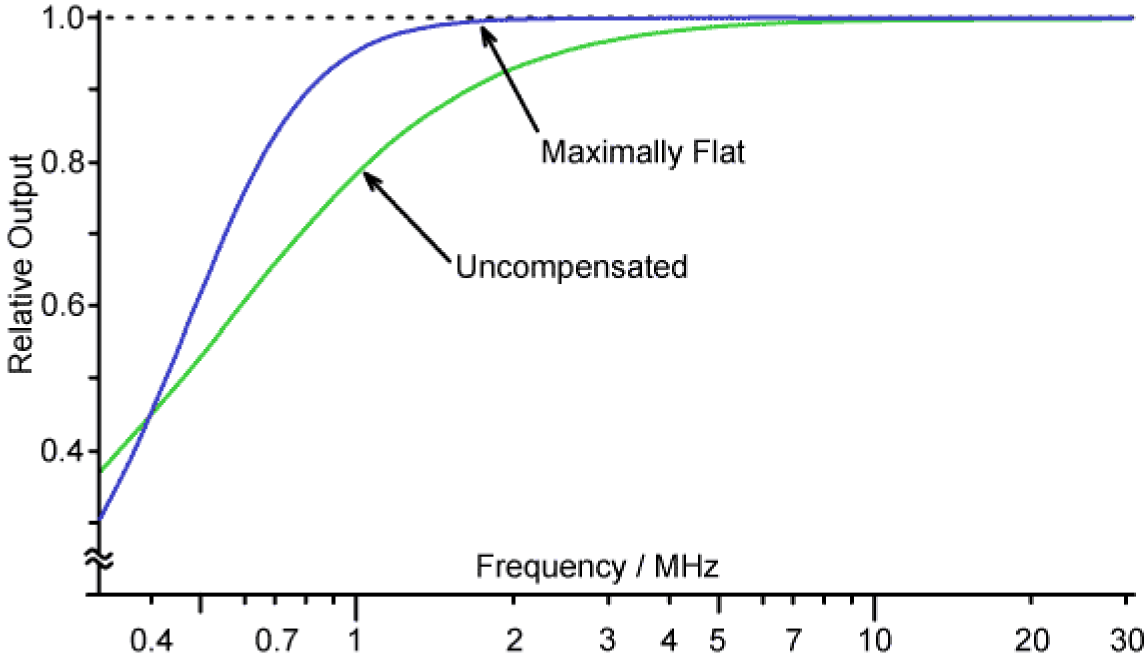
$f_x = 1/ [2\pi \sqrt{(L_i C_h)}]$	**5**

Below this frequency, the calculation formula for φ is:

$\varphi = [\mathrm{Arctan}\{ R_i / (X_{Ch} + X_{Li}) \}] + 180°$

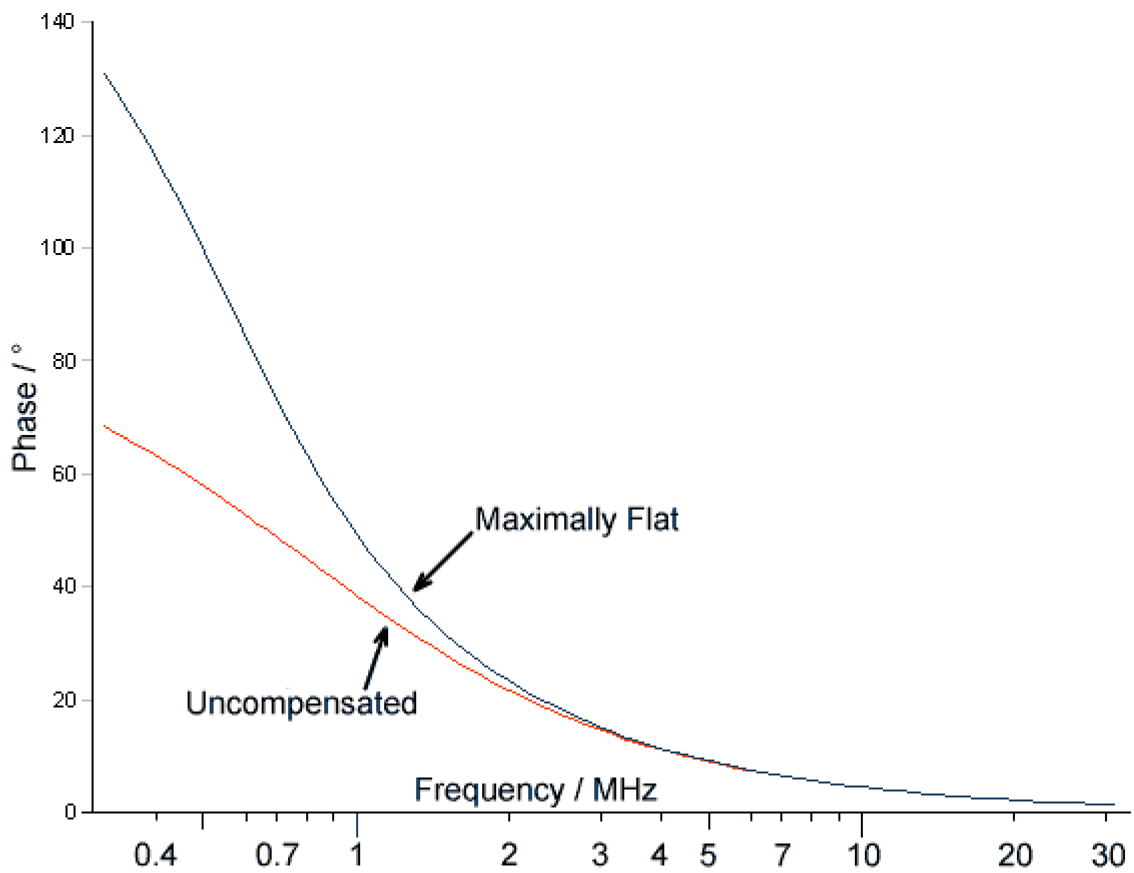
Shown below are graphs of the results from calculations for the case when $L_i = 10$ µH and $R_h = 50\ \Omega$. The compensation capacitance is given by equation (**3**) as:

$C_h = 2 L_i / R_i^2 = 8$ nF



Relative output vs. frequency for maximally flat current transformer

$L_i = 10\ \mu H$, $R_i = 50\ \Omega$, $C_h = 8$ nF. (Spreadsheet calculation: **maxflat_sim.ods**).



Phase vs. frequency for maximally flat current transformer

$L_i = 10\ \mu H$, $R_i = 50\ \Omega$, $C_h = 8$ nF. (Spreadsheet calc: **maxflat_sim.ods**).

In practice, the phase of the transformer output (relative to the input current) will usually become slightly negative at high frequencies due to transformer propagation delay and other parasitic effects, but the presence of C_h does not preclude the use of the various HF neutralisation techniques available[5].

Amplitude frequency-response measurements for a practical maximally flat current transformer are given in a separate article[6] and agree with the theory. Hence the circuit offers a possible solution to the long-standing secondary inductance problem. The example case with $L_i = 10$ µH and $R_i = 50\ \Omega$ gives an output that has dropped by just less than 1% at 1.5 MHz (compared to 11.7% for the uncompensated version) and, although this performance might be degraded in practice by other causes of amplitude error, the circuit is obviously a good candidate for the construction of magnitude bridges and accurate RF ammeters; i.e., for devices that discard the phase information in the output by rectifying the signal before using it. The properties of the network are a little less compelling for the design of ordinary impedance bridges, since moderate variations of sensitivity are usually of little consequence; but it is nevertheless possible to derive a companion voltage sampling network with identical phase performance[7], which might be of use to those who wish to make accurate SWR or return-loss analysers.

DWK, 2008, 2014

5 **Evaluation and optimisation of current transformer bridges**. D W Knight. http://g3ynh.info/zdocs/bridges/
6 **Amplitude responsof conventional and maximally-flat current transformers**. D W Knight.
7 **Resistive voltage-sampling and maxflat transmission bridges**. D W Knight.