

THE DECIBEL AND THE NEPER

Admiralty Handbook of Wireless Telegraphy, B.R.230 (Vol. II). HMSO London, 1938., Appendix "A".

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Updated, corrected, and references added by D W Knight. (Version 1.01, 19th Oct. 2010)

(Units are changed to use camelcase, e.g., db becomes dB, etc.).

1. Historical. - The "deciBel" is the 1/10th part of a "Bel" (after Alexander Graham Bell, inventor of the telephone sounder), a unit in which one may express power ratios, and gain or loss ratios of related quantities such as current and voltage. It originated in line telephony in 1923, when the American Telephone and Telegraph Company introduced a new unit, then called a "transmission unit"; this was to replace an older conception based on a ratio comparison between the decrease in signal strength produced by the given telephone line, and that produced by a "mile of standard cable" ^{1 2}. In 1924, an international advisory committee on long distant telephony in Europe, together with the representatives of the Bell system, agreed to recommend their countries to adopt as standards

EITHER the "**Bel**", a unit based on logarithms to the base 10, and equal to 10 of the American company's "transmission units",

OR the "**Neper**" (after Napier), a unit based on Napierian logarithms to the base e.

The growth in popularity of the deciBel, since 1929, has been so great that it is now almost a household word throughout all branches of Electrical Engineering and Acoustics.

2. Definitions

THE DECIBEL :- Two powers P_1 and P_2 are said to differ by N "Bels" when -

$$\frac{P_1}{P_2} = 10^N \quad \text{i.e.} \quad N = \text{Log}_{10} \frac{P_1}{P_2} \quad \dots \dots \dots \text{ in Bels}$$

Or in words - "The logarithm to the base 10 of the ratio of the powers, gives the gain or loss in Bels." If $P_1 = P_2$, then $N = 0$.

In practice, a unit of one Bel is found to be inconveniently large, and the 1/10th part of it - the deciBel - is more often used. Using the smaller unit we have -

$$\frac{P_1}{P_2} = 10^{0.1N} \quad \text{or} \quad N = 10 \text{ Log}_{10} \frac{P_1}{P_2} \quad \dots \dots \dots \text{ in deciBels}$$

The basic power ratio is $10^{0.1}$, that is, 1.259.

1 "**The Transmission Unit and Telephone Transmission Reference Systems**", W H Martin. Bell System Technical Journal. 3(3), July 1924. p400-408. Available from <http://bstj.bell-labs.com/>.

2 "**Decibel - The Name for the Transmission Unit**", W H Martin. Bell Sys. Tech. Journal. 8(1), Jan. 1929. p1-2.

THE NEPER : - Two powers P_1 and P_2 are said to differ by N "Nepers" when -

$$\frac{P_1}{P_2} = (e^2)^N$$

Therefore

$$(P_1/P_2)^{1/2} = e^N \quad \text{i.e.} \quad N = \frac{1}{2} \text{Log}_e(P_1/P_2) \quad \dots \dots \dots \quad \text{in Nepers}$$

Any ratio in dB may be readily converted to Nepers, for example 60 dB -

We have $\text{Log}_{10}(P_1/P_2) = 6$ Bels

Therefore

$$\frac{1}{2} \text{Log}_e(P_1/P_2) = 2.3026 \times 6 \times \frac{1}{2} = 6.907 \text{ Nepers}$$

The Neper is used in some European countries, but is less commonly encountered than the deciBel.

3. Cables, Amplifiers and Attenuators. - With these units, if the signal strength of a cable signal is 1/10th of that at the transmitting end, the loss is 1 Bel. With two similar cables in series, the received signal would be 1/100th of the transmitted one, and the loss would be two Bels.

In any amplifier, if the output power is 100 times the input, the "gain" is two Bels or 20 dB ; with two such amplifiers used in series, the gain in power ratio would be 10 000 : 1, or 40 dB.

One of the many advantages of the deciBel is that the enormous ranges of power involved in communication work can be expressed in figures conveniently small, instead of astronomically big ones.

In the above example, it will be noted that the nett power ratio of two amplifiers in series, involves the *product* of their individual power ratios, but only the *sum* of their deciBel equivalents. This is due to the logarithmic nature of the unit employed.

An attenuator may be regarded as the converse of an amplifier, and its power loss may be described in similar units. If two stages of amplification, one of p and the other of q dB is followed by a line having an attenuation of s dB, the nett level at the output of the system will be (p + q - s) dB above the original input level.

4. Voltage Gain in dB. - In general, two powers P_1 and P_2 will be compared by observing, either, the voltage developed across a given impedance, or, the current through it. **If the input and output impedance of (say), an amplifier are equal,** the power ratio will be proportional to the square of the voltage (or current) ratio. Therefore:

$$P_1/P_2 = (V_1/V_2)^2 = (I_1/I_2)^2$$

$$\text{i.e.} \quad N = 10 \text{Log}_{10}(V_1/V_2)^2 = 20 \text{Log}_{10}(V_1/V_2) \quad \dots \dots \dots \quad \text{in dB}$$

$$\text{or} \quad N = 20 \text{Log}_{10}(I_1/I_2) \quad \dots \dots \dots \quad \text{in dB}$$

The voltage or current ratio relation is very frequently misused to describe a power ratio, without regard to the necessary condition of equality between the input and output impedances. For example, it is not necessarily correct to say that an amplifier with a V.A.F. of 100, has a power gain

of 40 dB, when its "voltage gain" is given by that figure. Under proper conditions, however, an amplifier having a V.A.F. of 1000, or a power ratio of 1,000,000, would have a power (or voltage) gain of 60 dB.

Power and Voltage Gain Ratios Expressed in Decibels.

dB	Power ratio	Voltage ratio	dB	Power ratio	Voltage ratio
0.1	1.023	1.012	6.0	3.98	1.995
0.2	1.047	1.030	7.0	5.01	2.238
0.3	1.072	1.035	8.0	6.31	2.456
0.4	1.096	1.047	9.0	7.94	2.663
0.5	1.122	1.059	10.0	10.0	3.162
0.6	1.148	1.077	20.0	100	10.0
0.7	1.175	1.084	30.0	1000	31.62
0.8	1.202	1.096	40.0	10 000	100
0.9	1.230	1.109	50.0	100 000	316.2
1.0	1.259	1.122	60.0	1000 000	1000
2.0	1.585	1.259	70.0	10×10^6	3.162×10^3
3.0	1.995	1.412	80.0	100×10^6	10^4
4.0	2.510	1.585	90.0	1000×10^6	31.62×10^3
5.0	3.160	1.778	100.0	$10\ 000 \times 10^6$	10^5

Interpolation is easy ; for example, 15.2 dB = 10 + 5 + 0.2, which gives a power ratio $10 \times 3.16 \times 1.047 = 33.2$.

5. Absolute Power Above an Arbitrary Datum - deciBel Meters. - It is clear that the deciBel is only a relative unit of power level ; it is a ratio. If it is possible to arrive at some standard power as a "zero level", or datum of reference, then the absolute power may be expressed in deciBels above the chosen datum. Various datum levels have been chosen for different purposes, and 1 milliWatt, 6 milliWatts, and 12.5 milliWatts have been used in many cases. The zero output level of 1mW is frequently chosen for telegraphy work in radio engineering. In some recent sensitivity tests on a Service receiver, an output of 1mW at the telephone terminals, when using two stages of note magnification, was taken to correspond to a good R9 signal at the detector valve. In the same tests, input voltages between aerial and earth terminals were expressed in dB above 1 microVolt as datum (ref. R.30).

A receiver output power meter, is essentially an A.C. voltmeter, calibrated in deciBels with reference to the power passing into a load of definite impedance, contained within the instrument and replacing the normal one. With an impedance of (say) 1000 Ohms and a zero level of 1mW, the voltage to be measured would be equivalent to

$$\sqrt{(0.001 \times 1000)} = 1 \text{ Volt RMS}$$

For modern broadcasting telephony receivers, zero level in most output meters is fixed at 50mW. This is held to represent the weakest output to have any loudspeaker programme value. Scales are usually marked in mW and dB ; powers less than 50mW would be given negative readings in dB. For a maximum scale reading of 4000mW the power ratio is 80, and the point would therefore be marked 19.0 dB. In good instruments, the impedance may be adjusted to match the valve.

6. The Human Ear Justifies the deciBel. - The adoption of a new and unfamiliar unit to describe power ratios was not simply due to a desire to substitute small numbers for big ones; the origin of its size and logarithmic nature may both be traced to the peculiarities of the human ear. The power used by electrical apparatus employed in wireless, or other signalling services, is closely related to the power in the form of sound which it usually produces. From this inter-dependence between the twin sciences of acoustics and electrical signalling, it may readily be understood that the units employed by the one should be adjusted partially to fit the requirements of the other. The deciBel is a particularly convenient unit for measuring ratios of sound intensities.

7. The Unit Power Ratio and the M.S.C. - Prior to 1923, in telephony practice the M.S.C. (mile of standard cable) was defined as "the difference in loudness in an 800 cycle note, perceived by an observer comparing the notes from two equal telephones, one at the input and the other at the output end of a mile of standard telephone cable". The observed difference in loudness corresponds very nearly to a difference in power of 20%, and, from this, the suggestion arose to use a standard power ratio step of $10^{0.1}$; this gives a difference of $10^{0.1} - 1$, that is, about 25.9%. This percentage change in the loudness of a monotone is an easily audible one; it is, however, only slightly greater than the minimum audible change of intensity. It needs a well-trained ear to detect a change in loudness of as little as 10%, which is usually regarded as the lowest figure. Indeed, for most acoustic purposes, the deciBel is defined as the least ratio of sound intensities that can be appreciated by the human ear.

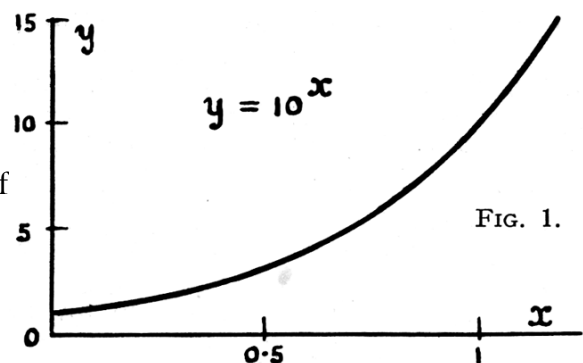
8. Logarithmic Response of the Human Ear - Fidelity Curves. - An absolute measure of the intensity of sound of a pure tone travelling in air or other medium, can be obtained in various ways and expressed in terms of

- (a) the R.M.S. pressure in dyne/cm² produced on a diaphragm placed in the path of the waves, or
- (b) the amplitude of the oscillation in cm, or
- (c) the number of Watts or milliWatts crossing an area of 1cm² at right angles to the direction of propagation.

If the loudness of a tone is increased, the amplitude increases, and, also, the rate of dissipation of energy in the form of sound.

It is found that, no matter what may be the absolute value of the intensity of any sound, the sensitivity of the ear is such that the same percentage change of the original intensity will always produce the same relative alteration of the "loudness sensation" at the ear. If an increase of intensity by 10 times produces a given effect at the ear, a further increase of 10 times, that is to say a total of 100 times, will produce twice the effect at the ear. Mathematically, this implies that the ear responds logarithmically to sounds of different intensities (i.e., powers), or that a logarithmic graph will be obtained if "sound intensity" is plotted along the y-axis, and "sensation of loudness" plotted along the x-axis. The curve is similar to that obtained on plotting $y = 10^x$, Fig. 1. In a graph of this nature, equal percentage increases in the value of y give equal increments along the x-axis. Considering the output of wireless receivers, the justification of the deciBel notation is that a change in output from 8 to 40mW (7dB) would seem to the ear identical with the change from 40 to 200mW, since the power ratio is again 5:1.

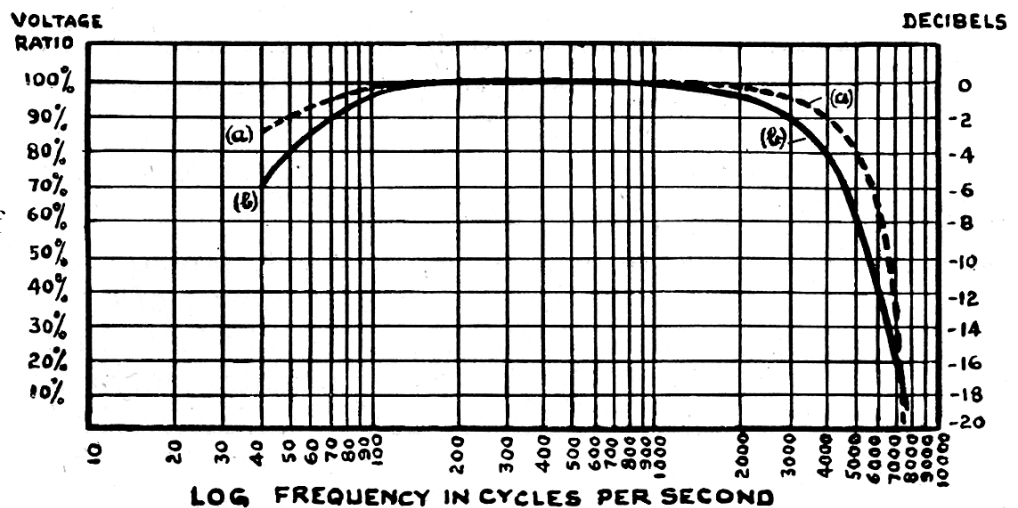
It is interesting to note that, in addition to the aural



response to power being logarithmic, the ear also responds logarithmically with respect to pitch (i.e., frequency). The graph of frequency plotted against "musical interval" along the x-axis would be similar to that of Fig. 1. Tones separated by an octave have a frequency ratio of 2 and are detected by the ear as being similar musical intervals. The various C's on a piano would appear equally spaced along the x-axis of a graph like that of Fig. 1.

Fig. 2 shows an amplifier fidelity curve. It represents the differential amplification of all frequencies in the usual AF range. Ideally, an amplifier should amplify all frequencies equally well, and, in that case, the fidelity curve would be a straight line. This curve shows that with

reference to the output at 500c/s, frequencies below 200c/s are not equally amplified. The graph gives a true representation of the fidelity with which the amplifier treats the various frequencies. At one time, these graphs were plotted using a voltage ratio scale along the y-axis ; since the ear is logarithmic in action, curves of that nature cannot represent truly



FREQUENCY-RESPONSE CURVES OF AN AMPLIFIER

(a) IN DECIBELS
(b) IN VOLTAGE RATIOS

FIG. 2.

the aural effect, and give a wrong impression of the performance of the apparatus. For example, at 4000c/s, the full curve shows the voltage ratio as being 80% or 1.25 taking the reciprocal ; corresponding to this voltage ratio, from the table we obtain approximately 2 dB, and the dotted curve accordingly passes through the point -2 dB.

9. Power in Sounds - deciBel Level - Tone Control. - The sensitivity of the ear varies with the frequency and also with the "level" at which the sound is produced. For any normal person there is a minimum sound intensity for each frequency, below which nothing is heard. Fig. 3 is due to Fletcher and Munson³; the lower curve shows the relative variation in level of the "threshold of audibility" over the ordinary AF band. The level at 1000c/s is arbitrarily marked 0 and is taken as a zero or datum level.

Ordinary conversation is approximately 60dB above the threshold of audibility. The curves may be termed "equal loudness curves", and show the relative insensitivity of the ear to low notes when the intensity level is low. In the case of the curve 0, a just audible tone at 100c/s has a sound Power about 38dB above a similar one at 1000c/s.

At an intensity level of 60dB at 1000c/s, an equally loud tone at 100c/s would only differ in power by about 12dB. For very intense sounds over about 90dB, the ear appears almost uniformly sensitive over the above frequency range. Still more intense sounds are felt rather than heard, and there is, in fact, a boundary called the "threshold of feeling". From the practical point of view it

3 "Loudness, Its Definition, Measurement and Calculation". H Fletcher and W A Munson, Bell System Technical Journal, 12(4), Oct. 1933. p377-430. Available from <http://bstj.bell-labs.com/>.

follows, that a voice cannot sound natural unless it is reproduced at its natural level. When this is not possible, or desirable, the volume control is turned up and the intensity level is raised; although the relative power content may remain the same, the increased sensitivity of the ear to the lower notes produces the well-known and unpleasant sensation of "booming". In a large auditorium, a surfeit of low tones masks the higher ones and impairs the general intelligibility of speech. For this reason, in large public address systems, a volume control should always be operated in conjunction with a "tone control".

It will be noted that the 50dB curve is the one which is flattest over the greatest range of frequencies, and it is sometimes considered that this is the optimum level of reproduction. Sound powers have, so far, been described in relative terms. It is, however, possible to put each sound power on an absolute basis. It has been measured that the average power of ordinary speech is somewhere between 10 and 15 microWatts ; on this basis G.W.C. Kaye calculated that the continuous talk of a Wembley Stadium football crowd of 100 000 persons represented only enough energy to light a small electric lamp

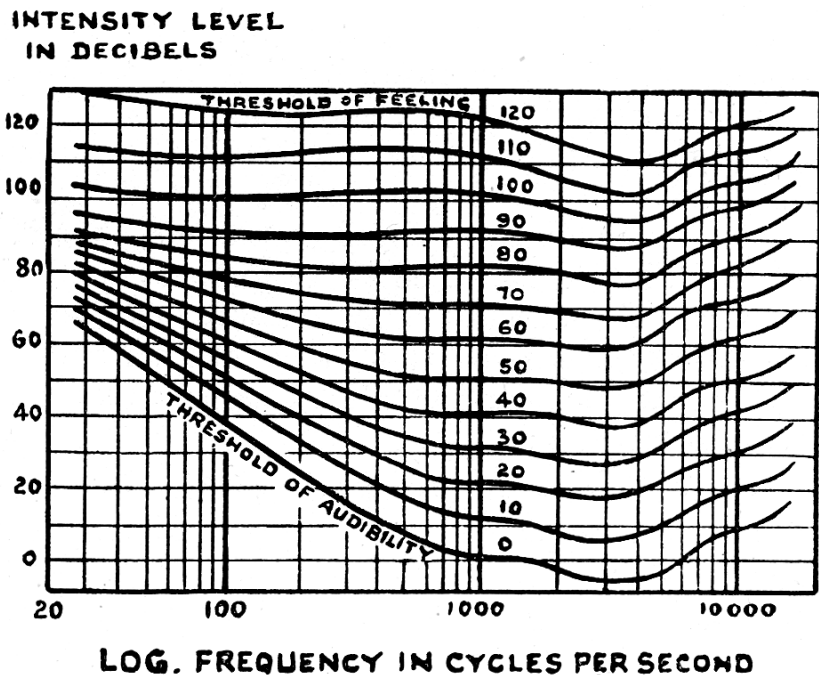


FIG. 3.

throughout the period of the game. It was similarly calculated that the acoustic disturbance created by a ship's siren amounts to about 6 microWatts per square centimetre at a distance of about 115 ft. This represented a total dissipation of energy by sound of about 1/3rd of a horse power. The peak power of the loudest sound in conversation has been stated to be of the order of 5000 microWatts, the power of the faintest sound being in the region of 0.01 microWatts ; this represents a range of about 57dB. The power corresponding to the datum line of audibility itself has been variously estimated by different observers. Although there is no general agreement about the exact figure, the number 10^{-16} Watts per square centimetre, for a free progressive wave of frequency 1000 cycles per second, is now usually quoted by American Research workers as the datum line. With this level as a basis, the level of ordinary conversation appears as about 100dB (not 60dB as quoted above), assuming the average power of ordinary speech to be 10 microWatts.

Musical sounds have a much greater power range than speech sounds. The peak power produced by a large orchestra may be of the order of 100 watts, and the noisiest drum can produce peaks of power of 25 watts.

Noises may, similarly, be expressed in terms of deciBels above a datum. Measurements have been made over a complete scale of noise levels, from that of an aeroplane engine (110dB) including in decreasing order of loudness, a roaring lion (90dB), an average office (45dB), the purring of a cat (about 15dB), and the noise in an underground vault in N.Y. city (10 to 15dB).

Man-made noise and static of various kinds is usually present as a background during the reception of broadcast telephony signals. In order that any transmission should provide good programme value for the ordinary listener, a committee of the I.E.E. recently decided that the signal/noise ratio should be not less than 40dB.

10. The Loudness Unit. The British Standard Phon. - The decibel serves as a useful unit for the expression of power ratios, but for the measurement of the relative loudness of sounds and noises, a new but related unit, the "phon", was introduced by the British Standards Institute in 1936.

The disadvantage of the deciBel as a loudness unit is demonstrated by Fig. 3, a different deciBel level being required to express equality of loudness at different frequencies in the AF range. A unit is needed with which to describe relative loudness, and to express equality of loudness along each of the equal loudness contours. It was decided to express loudness in terms of the equivalent loudness of a standard reference tone, and to define a datum of equivalent loudness based upon the R.M.S. pressure at the threshold of audibility of a free progressive tone at 1000 cycles per second; that R.M.S. pressure has been standardised in this country at 0.0002 dynes per square centimetre. More accurately, it is an intensity level of 10^{-16} Watts per square centimetre, corresponding to a sound pressure of 0.000204 dynes per square centimetre, at 76 centimetres of mercury and 20° Centigrade.

Loudness may be expressed as a ratio above this arbitrary datum, **the loudness of a sound in "phons" being numerically equal to the sound intensity in deciBels of an equally loud 1000 cycles per second pure note.** The equal loudness contours of Fig. 3 could be given labels in phons equal numerically to the deciBel levels of each at 1000 cycles per second.

In general, the loudness increases regularly with the intensity of the sound, within the limits of audibility, but, unfortunately, loudness is a sensation, the valuation of which depends upon subjective considerations such as the way in which the sound is heard, the person making the observations, etc. For example, in practice it is extremely difficult to say exactly when two sounds, such as that of a given continuous noise and a multiple of the standard reference tone, are equally loud; in general, it is found relatively easier to decide when one sound is very slightly louder than the other.

A hypothetical "normal" observer can make an accurate decision as to equivalent loudness if "he" listens with both ears to the source of sound and the reference tone, both presented alternately from a position directly in front of the observer. A satisfactory approximation to this hypothetical being is achieved by taking the average of the subjective decisions of a group of 10 observers; it has been estimated that such a group can arrive at a decision with an accuracy of ± 2 phons.

Subjective measurements of the above kind can usually only be made in a laboratory, and, for noise research work in engineering, it is very desirable to eliminate the personal equation as far as possible, by measuring noise by purely instrumental means. **Noise meters** usually consist of a microphone, amplifier, and indicating instrument; they can be supplied with circuits enabling them to approximate in function to the human ear, and to give direct **objective measurements** of loudness which are accurate when applied to pure tones.

These meters may also be applied to the measurement of the loudness of continuous noises which vary in nature from pure tones to "multi equal loudness" tones. With complex noises, the reading of the meter may be seriously in error, and satisfactory relative measurement of noise can only be achieved if it is previously calibrated in a laboratory, by some subjective equality method; with reproducible noises this is usually possible.

Objective noise meters are particularly valuable in dealing with noises of short duration, when there would not be time for any subjective measurement. Meters may be adapted for this purpose by the incorporation of suitable time constant circuits which conserve the energy produced during the sound impulse, permitting the meter to record a peak value from which it slowly falls.

In terms of phons the threshold of feeling has been estimated at 130 ; the noise level in motor-cars of various types has been estimated to range between 70 and 90 phons.

